ON THE AVERAGES OF THE DIVISORS OF A NUMBER

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Let n be some integer and let

$$n = p_1^{\alpha_1} \cdots p_r^{\alpha_r}$$

be its decomposition into prime factors. As usual we shall denote the *number* of divisors of n by $\nu(n)$ and the *sum* of the divisors by $\sigma(n)$ so that

$$\nu(n) = (\alpha_1 + 1)(\alpha_2 + 1) \cdots (\alpha_r + 1),$$

and

$$\sigma(n) = \sum_{d/n} d = \frac{p_1^{\alpha_1+1} - 1}{p_1 - 1} \cdots \frac{p_r^{\alpha_r+1} - 1}{p_r - 1}.$$

The various means of the divisors may be computed easily from the formulas that follow. For the *arithmetic mean* one has

$$A(n) = \frac{\sigma(n)}{\nu(n)} \tag{1}$$

and for the *geometric mean* the result is

$$G(n) = \sqrt[\nu(n)]{\prod_{d/n} d} = \sqrt{n}. \tag{2}$$

Finally the harmonic mean is defined by

$$\frac{1}{H(n)} = \frac{1}{\nu(n)} \cdot \sum_{d/n} \frac{1}{d}$$

and since

$$n \sum_{d/n} \frac{1}{d} = \sum_{d/n} \frac{n}{d} = \sum_{d/n} d$$

it follows that

$$H(n) = \frac{n\nu(n)}{\sigma(n)}. \tag{3}$$

The combination of the three formulas (1), (2), and (3) gives

$$H(n) \cdot A(n) = n = G(n)^2;$$

in other words, *the geometric mean of the divisors is the geometric mean of the arithmetic and harmonic means of the divisors.*

One may now ask in which cases these means may be integers. For the geometric mean this is trivial: *The geometric mean of the divisors is an integer only for square numbers.*

For the other two means the problem is by no means simple. We may observe first that both the arithmetic and harmonic means have the multiplicative property

$$A(a\cdot b) = A(a)\cdot A(b), \qquad H(a\cdot b) = H(a)\cdot H(b)$$

provided a and b are relatively prime. Thus one may only look for the *primitive* integral means, that is, such numbers that they are not the product of relatively prime factors, each of which has an integral arithmetic or harmonic mean.

Let us consider the arithmetic mean in a few special cases. For an odd prime p one has

$$A(p) = \frac{p+1}{2}$$

and this is always integral, while $p=2$ is an exception since $A(2)=3/2$ is not integral. From the multiplicative property we conclude that every number which is the product of different odd primes has an integral arithmetic mean of divisors while in the case of different prime factors including 2 the mean is only integral if one of the primes is of the form $4k-1$.

One may consider also when the powers of a prime may have an integral arithmetic mean of divisors. Since

$$A(p^\alpha) = \frac{p^{\alpha+1}-1}{(p-1)(\alpha+1)},$$

this is a problem closely connected with the solution of the congruence

$$x^k \equiv 1 \pmod{k}.$$

We shall not go into details about this problem. In certain cases the arithmetic mean is integral; for instance,

$$A(5^3) = 39, \qquad A(5^5) = 651.$$

It can be shown that no power of 2 can have an integral arithmetic mean.

The corresponding problems for the harmonic mean seem more interesting. In certain simple cases it can be established easily that the harmonic mean of the divisors cannot be integral. We mention first:

For the power of a prime the harmonic mean is not integral.

If namely $n=p^\alpha$ then

$$H(p^\alpha) = \frac{p^\alpha(\alpha+1)}{p^\alpha + p^{\alpha-1} + \cdots + p + 1}.$$

Here p^α is relatively prime to the denominator and since $\alpha+1$ is a number less

than the denominator the harmonic mean cannot be integral.

Another observation is:

When $n \neq 6$ is the product of different prime factors the harmonic mean cannot be integral.

We write

$$n = p_1 p_2 \cdots p_r$$

where the primes are arranged in increasing order, and one finds

$$H(n) = \frac{p_1 p_2 \cdots p_r}{(p_1+1)\cdots(p_r+1)} \cdot 2^r.$$

Let us assume first that n is odd, and let us write

$$H(n) = \frac{p_1 \cdots p_r}{\frac{p_1+1}{2} \cdots \frac{p_r+1}{2}}.$$

Here the denominator contains at least r prime factors, and since p_r is not among them the quotient cannot be integral. Next let $p_1 = 2$ so that

$$H(n) = \frac{2^{r+1} \cdot p_2 \cdots p_r}{3 \cdot (p_2+1)\cdots(p_r+1)} = \frac{4 \cdot p_2 \cdots p_r}{3 \cdot \frac{p_2+1}{2} \cdots \frac{p_r+1}{2}},$$

and this expression can only be integral if $p_2 = 3$ so that

$$H(n) = \frac{2p_3 \cdots p_r}{\frac{p_3+1}{2} \cdots \frac{p_r+1}{2}}.$$

Since none of the $r-2$ factors in the denominator are equal to p_r we conclude that each of them must be equal to one of the other prime factors in the numerator. But the assumption

$$\frac{p_3+1}{2} = 2$$

or $p_3 = 3$ was already excluded.

A result of some interest is the following:

A perfect number has an integral harmonic mean of divisors.

Since a perfect number is defined by the property that

$$\sigma(n) = 2n$$

it follows that

$$H(n) = \frac{n\nu(n)}{\sigma(n)} = \frac{\nu(n)}{2}.$$

When n is an even perfect number it has the form $n=2^{\alpha}\cdot p$ so that $\nu(n)=(\alpha+1)\cdot 2$ and $H(n)=\alpha+1$ is integral. When we have an odd perfect number it is known that one of the exponents in the prime factor decomposition is odd so that $\nu(n)$ is even also in this case.

By means of the tables of J. W. L. Glaisher giving the values of $\sigma(n)$ and $\nu(n)$ one can determine rather easily the values of n below 10000 for which $H(n)$ is integral. One finds the following table:

TABLE I

n	$H(n)$	$A(n)$
$6=2\cdot 3$	2	3
$28=2^2\cdot 7$	3	$9\frac{1}{3}$
$140=2^2\cdot 5\cdot 7$	5	28
$270=2\cdot 3^3\cdot 5$	6	45
$496=2^4\cdot 31$	5	$99\frac{1}{3}$
$672=2^5\cdot 3\cdot 7$	8	84
$1638=2\cdot 3^2\cdot 7\cdot 13$	9	182
$2970=2\cdot 3^3\cdot 5\cdot 11$	11	270
$6200=2^3\cdot 5^2\cdot 31$	10	620
$8128=2^6\cdot 127$	7	$1161\frac{1}{7}$
$8190=2\cdot 3^2\cdot 5\cdot 7\cdot 13$	15	546

It would be of interest to make a systematic examination of such numbers up to higher numerical limits. This requires an extension of Glaisher's Number-divisor tables, a project which would be desirable also in connection with other number theoretical investigations.

There are, however, various methods by means of which one can construct new numbers with integral harmonic means from given ones. Let us suppose that n has this property so that

$$n\cdot\nu(n) = a\cdot\sigma(n), \qquad a = H(n).$$

We multiply both sides by some number k relatively prime to n and write $n_1=k\cdot n$. Then one finds

$$H(n_1) = \frac{n_1\cdot\nu(n_1)}{\sigma(n_1)} = \frac{a\cdot k\cdot\nu(k)}{\sigma(k)}.$$

Thus if the right-hand expression is an integer the number n_1 will also have an integral harmonic mean. This may again be examined by means of Glaisher's tables. The values of a in the last column of the table given above lead successively to the following numbers:

TABLE II

n	$H(n)$	n	$H(n)$
$2\cdot 3^2\cdot 7\cdot 13\cdot 17$	17	$2^3\cdot 3^2\cdot 5^2\cdot 31\cdot 53$	53
$2\cdot 3^2\cdot 5\cdot 7\cdot 13\cdot 29$	29	$2^6\cdot 13\cdot 127$	13
$2^3\cdot 3\cdot 5^2\cdot 31$	15	$2^6\cdot 3^2\cdot 13\cdot 127^*$	27
$2^3\cdot 3^2\cdot 5^2\cdot 31$	27	$2^6\cdot 3^2\cdot 5\cdot 13\cdot 127$	45
$2^3\cdot 3^2\cdot 5^2\cdot 17\cdot 31$	51	$2^6\cdot 3^2\cdot 13\cdot 17\cdot 127$	51
$2^3\cdot 5^2\cdot 19\cdot 31$	19	$2^6\cdot 3^2\cdot 5\cdot 13\cdot 17\cdot 127$	85
$2^3\cdot 5^2\cdot 7^2\cdot 19\cdot 31$	49	$2^6\cdot 3^2\cdot 5\cdot 13\cdot 29\cdot 127$	87
$2^3\cdot 5^2\cdot 7^2\cdot 13\cdot 19\cdot 31$	91	$2^6\cdot 3^2\cdot 13\cdot 53\cdot 127$	53
$2^3\cdot 3\cdot 5^2\cdot 29\cdot 31$	29	$2^6\cdot 3^2\cdot 5\cdot 13\cdot 89\cdot 127$	89
$2^3\cdot 5^2\cdot 19\cdot 31\cdot 37$	37		

Table I shows that there are eleven numbers below 10,000 with integral harmonic mean for the divisors. Among these are the four perfect numbers below this limit. One verifies simply that an even perfect number cannot have an integral arithmetic mean of the divisors. However, if these are excluded from Table I the remaining numbers have the property that also their arithmetic means of divisors are integral. One might be led to the conjecture that this would be a general property and the computation of Table II was executed with this in mind. However, the number marked by an asterisk proved to be a counter example. It is not perfect and its arithmetic mean of divisors is not integral. A much more interesting conjecture, however, appears from our numerical computations, as an extension of the famous conjecture for perfect numbers, namely, that *a number with integral harmonic mean of divisors must be even.*

POLYGONS WITH TWO EQUIANGULAR POINTS

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1. Introduction. In 1816 A. L. Crelle [1] discovered that in every triangle ABC there are two points G and G' such that $\measuredangle GAB=GBC=GCA=G'BA=G'CB=G'AC=\omega$. After H. Brocard [2] rediscovered this feature in 1875, he and other mathematicians found a great number of related facts [3]. Since then the points G, G' and angle ω have been called Brocard's points and Brocard's angle. Interest in the subject remained at a high level for many years, and in 1886 R. Tucker [4] found that Moebius' harmonic quadrangle [5] has a similar property. Although this type of geometry of the triangle fascinated quite a few, all efforts failed to construct or calculate polygons with more than four sides having two equiangular points [6] as defined above. In 1930 K. Hagge [7] pub-

lished a lecture in which he gave a history of the problem, and hinted at a construction for one class of these polygons without giving any lead how he had arrived at his guess.

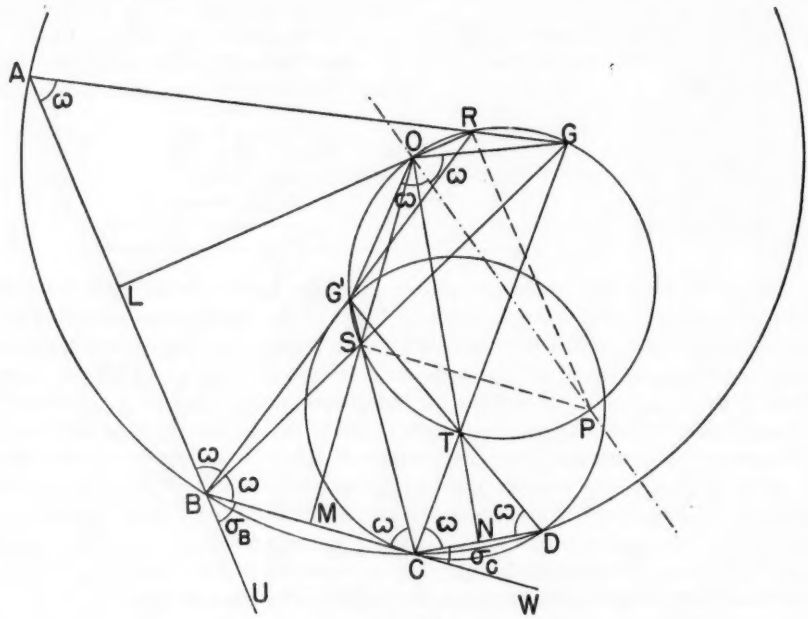

FIG. 1

2. Construction of nomogons. Let AB and BC in Figure 1 be two consecutive sides of the polygon to be found and their exterior angle $\sigma_B = CBU$. Then Brocard's angle ω determines two equiangular points [8] G and G' at the intersections of AR, BS and BR, CS, respectively. The direction of a third side CD of the polygon follows from the requirement that $\angle GCD = \omega$, and in order to make $\angle G'DC = \omega$, D has to be on a circle with ω as an inscribed angle intercepting CG'. By the same construction one vertex after the other could be found from the given two sides and two angles, and this rigid character of the polygons under discussion has caused me to call them nomogons.

While R and S are the vertices of isosceles triangles with bases AB and BC respectively, T is the vertex of isosceles triangle CTD. The intersection O of the perpendicular bisectors LR and MS of the sides AB and BC is the center of a circle passing through A, B, and C. Now, because $\measuredangle RG'S$, ROS, and RGS equal σ_B or $\pi - \sigma_B$, points O, G, G', R, and S are on a circle. Further, because $\measuredangle G'RG$, $G'SG$, and $G'TG$ equal 2ω or $\pi - 2\omega$, point T is on the same circle as the five other points. Let us assume that the perpendicular bisectors MS and NT intersect

in a point O' different from O; then $\measuredangle SG'T$, SGT, and $SO'T$ are equal to $\sigma_C = DCW$ or $\pi - \sigma_C$. Therefore O' is also on the same circle, and since it is not identical with either S or T, it is identical with O. It follows that CD and all following sides of the nomogon are chords to the circle through ABC, and every nomogon is an inscribed polygon. It will be shown below how it is determined whether or not the nomogon closes after one or more circuits of the circle.

The circle $OGG'RST\cdots$ corresponds to Brocard's circle in the triangle, and the other end P of its diameter through O is the counterpart of Lemoine's point [9]. Because $\measuredangle GSO = G'SO = \pi/2 - \omega$, $\angle G'OG = 2\omega$, and both the arcs and the chords OG and OG' are equal respectively; it follows that $\measuredangle POG = POG' = \omega$.

3. The circumcircle and inscribed conic. When the circumscribed circle, one equiangular ray GA, and Brocard's angle are given, the nomogon is readily found as a chain of chords. If A moves on the circle, these chords envelop a curve whose equation with O as origin, OP as positive axis of ordinates, $OA = r$, and $OG = f$, is found thus: Let the slope of lines AB and AG be θ and $\theta - \omega$, respectively. Since AG is a line through $G(f \sin\omega, f\cos\omega)$ and AB is a line through $A(x_A, y_A)$, their equations are

$$(y - f\cos\omega)\cos(\theta - \omega) = (x - f\sin\omega)\sin(\theta - \omega)$$
$$(y - y_A)\cos\theta = (x - x_A)\sin\theta$$

respectively. We use the first equation and the equation $x^2 + y^2 = r^2$ of the circumscribed circle as simultaneous equations to determine x_A and y_A, and substitute them in the second equation. Thus the equation of AB is

$$y = x\tan\theta + f\cos\omega \pm \frac{\sin\omega}{\cos\theta}\sqrt{r^2 - f^2\cos^2\theta.}$$

From $\partial y/\partial\theta = 0$ we find the parametric equations of the envelope:

$$x = \pm\frac{r^2\sin\omega\sin\theta}{\sqrt{r^2 - f^2\cos^2\theta}}, \qquad y = f\cos\omega \pm \frac{(r^2 - f^2)\sin\omega\cos\theta}{\sqrt{r^2 - f^2\cos^2\theta}},$$

and the elimination of θ produces the single equation

$$\frac{x^2}{r^2\sin^2\omega} + \frac{(y - f\cos\omega)^2}{(r^2 - f^2)\sin^2\omega} = 1. \tag{1}$$

This inscribed conic, defined as a feature of the triangle, is known as Brocard's ellipse. Its foci are identical with the equiangular points, but it is an ellipse only if these points are inside the circumcircle, that is, $f < r$. If $f > r$, it is a hyperbola.

By determining the intersections of the conic with the circumscribed circle one finds two double points:

$$x = \pm\frac{r}{f}\sqrt{f^2 - r^2\cos^2\omega}, \qquad y = \frac{r^2\cos\omega}{f} \tag{2}$$

which means that the inscribed conic is tangent to the circumscribed circle in two points. The points of tangency are real if $f>r\cos\omega$, they coincide if $f=r\cos\omega$, and they are imaginary if $f<r\cos\omega$. At the same time $OP=f/\cos\omega$ becomes greater than, equal to, or less than r, respectively.

From the two basic facts just proved that every nomogon is inscribed into a circle and that it is circumscribed about a conic which touches the circumcircle in two points, the complete solution of the problem of polygons with two equiangular points can be deduced. There are various chains of deductions, each offering a different aspect of our problem.

In the language of hyperbolic non-euclidean geometry and using the circumcircle as absolute, the inscribed conic may be (**A**) a proper circle, (**B**) a horocycle, (**Ca**) an ordinary equidistant curve, or (**Cb**) an ultra-infinite equidistant curve. Each of these circles generates a different class of nomogons which can also be defined by the position of the equiangular points with relation to the center of the circumscribed circle and to each other. For, if one considers f and ω variable, producing an infinite number of nomogons, the conditions $f=r$ and $f=r\cos\omega$ indicate those nomogons whose equiangular points are on the circumcircle and on a limiting circle passing through O and tangent to the circumcircle, respectively. If the equiangular points are on the circumcircle, both the inscribed conic and the nomogon degenerate into the same straight line, and so we have the following four classes of nomogons:

(**A**) G and G' inside the limiting circle, P inside the circumcircle, all sides of finite length.

(**B**) G and G' on the limiting circle, P on the circumcircle, the vertices of the nomogons approach one point.

(**C**) G and G' outside the limiting circle, P outside the circumcircle, the vertices of the nomogons approach two points;

(**a**) G and G' outside the limiting circle but inside the circumcircle, the vertices of the nomogons approach the two points from both sides without alteration;

(**b**) G and G' outside the circumcircle, the vertices of the nomogons approach the two points alternating from one side of each point to the other.

From the fact that $OP=f/\cos\omega$ it follows easily that Brocard's circle meets the circumcircle in the intersections of the axis with the absolute, that is, in two imaginary points in case (**A**), in a double point in case (**B**), and in two real points in case (**C**). On the other hand, the equation of the axis being $y=r^2\cos\omega/f$, we have $y\times OP=r^2$ and, therefore, Lemoine's point P is the center of the non-euclidean circles. Hence the vertices of the nomogon are produced by equal hyperbolic motions whose center is Lemoine's point P; the motion is a rotation in case (**A**), a parallel displacement in case (**B**), a translation in case (**Ca**), and a glide-reflection in case (**Cb**). In the last case the sides of the nomogon touch the two branches of the ultra-infinite equidistant curve alternately.

When four consecutive vertices $ABCD$ of the nomogon are given, the center P is found as Brianchon's point of the circumscribed hexagon $ACCBBD$. The following vertices of the nomogon are then readily constructed by making $\measuredangle DPE, EPF, \cdots = APB$ and, therefore, the nomogon may be described as a regular polygon whose consecutive sides are parallel.

Using the language of projective geometry, the circumcircle and the inscribed conic, because of their double tangency, may be projected simultaneously into concentric circles. Therefore, the tangents to the inscribed conic will cut projective non-involutory ranges on the circumcircle, and there will be a projectivity T on the circle transforming A to B, B to C, and so on. We have case (**A**), (**B**), or (**C**) according as projectivity T is elliptic, parabolic, or hyperbolic, and subclasses (**a**) or (**b**) according as it is direct or opposite, that is, preserves or reverses the sense.

4. The cross ratio k. The fact that the cross ratio of four consecutive vertices of a nomogon is constant, which follows from projectivity T, can be derived directly from the basic figure by a short trigonometric calculation. With α, β, γ designating inscribed angles with their vertex V on the circle $ABCD$ and intercepting AB, BC, CD, the Law of Sines expressed for GB in triangles GAB and GBC results in the formula

$$\cot(\beta+\gamma) = \cot\beta - \cot(\alpha+\beta) - \cot\omega \tag{3}$$

which allows us to calculate point D when besides A, B, and C, the angle ω is given, or vice versa. Eliminating ω from this formula taken for two consecutive quadruples, we get:

$$\cot(\gamma+\delta) + \cot\beta = \cot(\alpha+\beta) + \cot\gamma \tag{4}$$

which yields a fifth vertex without knowledge of ω. Formula (4) can readily be rewritten in the form,

$$\frac{\cot\beta - \cot(\beta+\gamma)}{\cot\beta + \cot\alpha} = \frac{\cot\gamma - \cot(\gamma+\delta)}{\cot\gamma + \cot\beta} \tag{5}$$

and this expression which has the same value k for every consecutive quadruple of vertices, can be written:

$$k = \frac{\sin(BVA)}{\sin(CVA)} : \frac{\sin(BVD)}{\sin(CVD)} = (BCAD) = (CDBE) = \cdots . \tag{6}$$

Thus we have the cross ratio

$$k = (BCAD) = \frac{\cot\beta - \cot(\beta+\gamma)}{\cot\beta + \cot\alpha} . \tag{7}$$

If solved for $\cot(\beta+\gamma)$, this formula reads:

$$\cot(\beta+\gamma) = (1-k)\cot\beta - k\cot\alpha \tag{8}$$

and by substituting $\cot\beta-\cot(\beta+\gamma)$ from formula (3) we find:

$$\cot\omega = k(\cot\alpha+\cot\beta)-\cot(\alpha+\beta). \tag{9}$$

These two formulas allow us to compute the fourth vertex and Brocard's angle, respectively, when three consecutive vertices are required to generate a nomogon characterized by a certain cross ratio k.

On the other hand, it follows from formula (9) that to every value of k there is a corresponding value of ω, and because it is also true that D is uniquely determined by cross ratio k, we conclude that every chain of chords with a constant cross ratio of its intercepting inscribed angles is a nomogon.

Among the nomogons with a certain cross ratio k, the one which has its equiangular points in the center of the circumscribed circle and which, therefore, always belongs to class (**A**), has equal sides; if in formula (7) we make $\alpha=\beta=\gamma=\alpha_0$, we find

$$k=\frac{1}{4\cos^2\alpha_0} \qquad \left(0\leqq\alpha_0\leqq\frac{\pi}{2}\right) \tag{10}$$

and since the number n of sides per circuit is

$$n=\frac{\pi}{\alpha_0} \tag{11}$$

the two formulas allow us to find the number n generated by a particular value $\infty>k>\frac{1}{4}$; or vice versa we can find the k resulting in a given number $2\leqq n\leqq\infty$ of sides per circuit. Remembering that both the cross ratio k and the number n of sides of a polygon are projective properties, we find that any triangle is characterized by $k=1$, any quadrangle by $k=\frac{1}{2}$, the two kinds of pentagons by $k=\frac{1}{2}(3\pm\sqrt{5})$, any hexagon by $k=\frac{1}{3}$, and so forth. If n is irrational or transcendent the nomogon does not close; if it is rational ($n=u/v$, u and v positive integers, $v\leqq u/2$), the nomogon closes after v circuits and has a total number u of sides until it closes. In the classes other than (**A**) the number of sides per circuit is infinite; yet, each nomogon is characterized by a definite value of k. This is the solution of the old problem of finding a nomogon with a required number of sides.

5. Some properties of the nomogons. It is now but a question of predilection how far one wants to investigate the properties of the nomogons, and a question of space how many of our results we may mention here.

(a) In Figure 1, the parallels RP, SP to the sides AB, BC through the vertices of the similar isosceles triangles ARB, BSC intersect in Lemoine's point P. Therefore, if one keeps AB, BC unchanged and varies ω and thereby k and the number of sides, the locus of P is a straight line through B such that

$$\cot ABP=\cot\alpha-2\cot(\alpha+\beta). \tag{12}$$

The fact that this line is identical with the so-called symmedian in the triangle

[10] and with the diagonal in Moebius' quadrangle explains why the latter can be used to construct the other [11].

(b) In Figure 2, O, P, G, G', A, B, and R have the same meaning as in Figure 1; AG is reflected in GO into $A'G$, and BG' is reflected in OP into $B'G$. Putting $\angle AGG'=\phi$, we find $\angle G'GA'=2OGA+AGG'=\pi-(2\omega+\phi)$ and $\angle G'GB'=GG'B=G'BU+G'UB=2\omega+\phi$; therefore, $A'GB'$ is a straight line while the segments $A'G$, GB' equal AG, $G'B$, respectively. With GP produced to K and

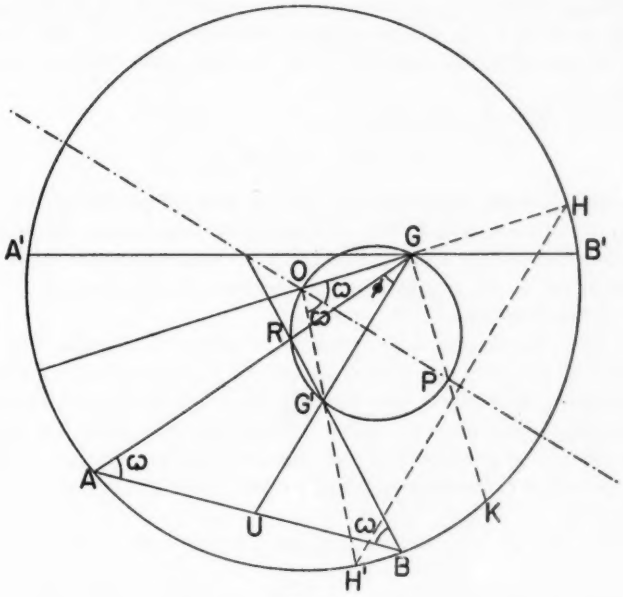

FIG. 2

$OG=f$ we get $AG\times BG'=\overline{GK}^2=-(f^2-r^2)$, and this equation states that the rectangle formed by the two equiangular rays associated with any one side of the nomogon equals the negative value of the power of the equiangular points with respect to the circumscribed circle, and is therefore invariant; it is independent of any other condition, including even angle ω and what it implies.

(c) The Law of Sines applied to triangles ABG and BCG' yields:

$$\frac{AG}{AB}=\frac{\sin(\alpha+\beta+\omega)}{\sin(\alpha+\beta)}; \qquad \frac{BG'}{BC}=\frac{\sin\omega}{\sin(\alpha+\beta)}. \tag{13}$$

We divide the product $AG\times BG'$ found from these formulas by the square $4r^2\sin^2\omega$ of chord HH' intercepted by OG and OG' produced, and find the value of the quotient,

$$(14) \qquad \frac{AG \times BG'}{HH'^2} = \frac{\cot \omega + \cot (\alpha + \beta)}{\cot \alpha + \cot \beta} = \frac{\cot \beta - \cot (\beta + \gamma)}{\cot \alpha + \cot \beta},$$

which equals k according to formula (7). Substituting $r^2 - f^2$ for $AG \times BG'$, we get

$$(15) \qquad k = \frac{r^2 - f^2}{4r^2 \sin^2 \omega},$$

a formula which defines the cross ratio k for a given position of the equiangular points with relation to the circumscribed circle and axis OP. For class (**A**) we find $\infty \geqq k > \frac{1}{4}$ as before, for class (**B**) $k = \frac{1}{4}$, for class (**Ca**) $\frac{1}{4} > k > 0$, and for class (**Cb**) $0 > k \geqq -\infty$.

(d) Formula (15) solved for f is

$$(16) \qquad f^2 = r^2(1 - 4k \sin^2 \omega).$$

This is the equation in polar coördinates with axis OP of the locus of the equiangular points for any given k. The pencil of curves covers the whole plane and, therefore, any two points equally distant from O can be chosen for equiangular points. The locus for the quadrangle is a lemniscate, for nomogons of class (**B**) it consists of two limiting circles.

(e) For the triangle and quadrangle it is known that $\cot \omega = \cot \alpha + \cot \beta + \cot \gamma$ and $\cot \omega = \frac{1}{4}(\cot \alpha + \cot \beta + \cot \gamma + \cot \delta)$, respectively. From formula (8), with α and γ exchanged, and formula (9) we eliminate $\cot(\alpha + \beta)$ and get $\cot \omega = k(\cot \alpha + 2 \cot \beta + \cot \gamma) - \cot \beta$. Let the nomogon be closed and its total number of sides be u as defined above; then we add u such equations, one for each of the u sets of three successive subtended angles, and divide by u. We find

$$(17) \qquad \cot \omega = \frac{4k - 1}{u} (\cot \alpha + \cot \beta + \cot \gamma + \cdots).$$

With $k = 1$, $u = 3$, and with $k = \frac{1}{2}$, $u = 4$ this general formula yields the above equations for the triangle and the quadrangle, and for the hexagon with $k = \frac{1}{3}$, $u = 6$, for instance, we find that the coefficient is 1/18.

6. Properties of nomogons explained by modern geometry. In the following paragraphs we propose to show another approach to our problem which explains the basic properties of the nomogons by the methods of modern geometry and goes far in deducing even their particular geometrical properties by such methods.

Similar to our procedure in Figure 1, we assume that in Figure 3 three vertices ABC and Brocard's angle ω are given. We produce AG and BG to A' and B', respectively, on the circumscribed circle, and find that arc $BA' = CB'$ because they are intercepted by equal inscribed angles. By subtracting the common part CA', we get $A'B' = BC$. We produce CG to C' on the same circle and rotate $B'C'$ about O back into CD. The angle of rotation $B'OC = 2B'BC = 2\omega$.

When the procedure is repeated, two polygons $ABC\cdots$ and $A'B'C'\cdots$ result which are (a) congruent, (b) perspective with center G, and (c) rotated about O by angle 2ω.

It has yet to be proved that (d) the inscribed polygons have a second equiangular point to make them nomogons, and also, in order to ensure complete generality, a proof is needed that (e) all vertices of the two nomogons are necessarily on the same circle. Locate N' on the circle so that $N'A'B'C'\cong ABCD$. Then, since $\angle BB'A'=BAA'=\omega$ and arc $BN'+N'A'=AB+BN'$, $\angle AA'N'=A'AB=\omega$. When triangle $A'B'G$ is rotated through the angle 2ω about O, it

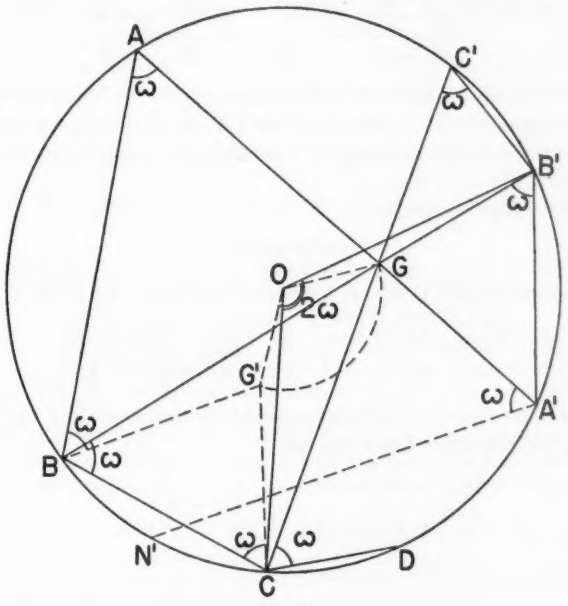

FIG. 3

will coincide with triangle BCG', and similarly the others. This proves not only the existence of the second equiangular point but also that $GO=G'O$ and $\angle GOG'=2\omega$. On the other hand, $\angle CC'B'$ which intercepts the same arc as $\angle COB'=2\omega$, equals ω only if C' is on the same circle as points C, B, and B'. Hence D and all following vertices of nomogon $ABC\cdots$ lie on circle ABC. The following discussions are based on the five facts thus proved.

The pairs of points AA', BB', $CC'\cdots$ are in involution because through projection from G point $A\to A'\to A$, $B\to B'\to B$, $\cdots$. The same correspondence is produced by a transformation T which transforms A into B, B into C, $\cdots$ and a rotation R by 2ω which transforms B into A', C into B', $\cdots$. Thus we have

$$I=RT,\ (RT)^2=1. \tag{18}$$

If RT is an involution I, $R^{-1}T^{-1}$ is also an involution I' with center G'. Thus the fact that every inscribed polygon with one equiangular point has another equiangular point is deduced from general principles.

Because both I and R are projective transformations, transformation T is also projective. Thus it is proved that the vertices of a nomogon are found by iterated linear transformation and that, therefore, the cross ratio of four consecutive vertices is constant. The period of transformation T determines the number of sides of the nomogon.

It is well known that the formula

$$\frac{w'-a'}{w'-b'} : \frac{c'-a'}{c'-b'} = \frac{z-a}{z-b} : \frac{c-a}{c-b}, \tag{19}$$

where all numbers are complex, and the cross ratio on each side is real, represents a linear transformation of a circle into itself when three pairs of points aa', bb', cc' are given; it becomes an algebraic expression of transformation T if we put $a'=b$, $b'=c$, and $c'=d$.

Rotation R may be written

$$w = w'e^{2i\omega} \tag{20}$$

and thus the product RT is found to be an involution ($\alpha+\delta=0$) if

$$e^{2i\omega} = \frac{b(d-c)(c-a) - a(d-b)(c-b)}{b(d-c)(c-a) - c(d-b)(c-b)}. \tag{21}$$

This formula allows us to calculate the angle ω corresponding to four consecutive vertices. On the other hand, we have

$$\frac{d-a}{d-b} : \frac{c-a}{c-b} = 1-k \tag{22}$$

and hence

$$d = \frac{c(a-b) + kb(c-a)}{(a-b) + k(c-a)} \tag{23}$$

which, substituted in formula (21), yields:

$$e^{2i\omega} = \frac{a(a-b)(c-b) + kb(c-a)^2}{c(a-b)(c-b) + kb(c-a)^2}. \tag{24}$$

Thus we are able to calculate a nomogon with a given number of sides when three consecutive vertices are given, and also to determine the angle ω which generates this nomogon. Note that formulas (21) to (24) accomplish the same as formulas (3), (7), (8), and (9).

Because involution I' with center G' is generated from involution I with center G by a rotation R^{-1}, it follows that $R^{-1}(G)=G'$. On the other hand, $RT(G)$

$=G$ implies that $T(G)=R^{-1}(G)$ and hence generates G'. Thus G is transformed into G' both by the euclidean rotation R^{-1} about O and by the non-euclidean rotation T about P. We conclude that G' is symmetrical to G with respect to axis OP and $\angle OPG=G'OP=\omega$. A transformation $RT(P)=P'$ determines a line PP' which, like any other line connecting a pair of conjugate points produced by means of involution I, passes through point G. On the other hand, $RT(P)=R(P)$ and therefore $PO=P'O$ and, considering that $\angle GOP$ has just been proved to equal ω, $\angle GOP'=\omega=GOP$. From the last three statements it follows that OGP and, of course, $OG'P$ are euclidean right angles and that the equiangular points G and G' are on a circle with diameter OP. It also follows that P is Lemoine's point.

Let K be one imaginary circular point at infinity. A line through K and $T^{-1}(K)$ is tangent to the inscribed conic. But because $R(K)=K$, we have $RT(T^{-1}(K))=K$ and, since a line through the conjugate pair $T^{-1}(K)$ and $RT(T^{-1}(K))$ passes through G, tangents to the conic from the imaginary circular points pass through G and G' and therefore, according to the usual projective definition, the equiangular points are foci of the inscribed conic.

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2. Nouvelles Annales de Mathématiques. Deuxième Série, Tome 14. 1875. Page 192. Question 1166: Trouver, à l'intérieur d'un triangle rectiligne ABC, un point O tel que les angles OAB, OBC, OCA soient égaux. Page 286. Solution by M. C. Chadu mentioning the second equiangular point and giving formulas for ω, the equiangular rays and so on.

3. Mostly in mathematical journals like: Comptes rendus de l'Association Française pour l'Avancement des Sciences, Mathésis, and many others as quoted in the Enzyklopädie der Mathematischen Wissenschaften. Vol. III.1.2.: Neuere Dreiecksgeometrie by G. Berkhan and W. Fr. Meyer, p. 1181.

4. Some properties of a quadrilateral in a circle, the rectangles under whose opposite sides are equal. Reprints from Educational Times, 1886. 44, 125–135.

5. (Crelles) Journal f.d. reine u. angewandte Mathematik. Vol. 52. 1856. Pages 239–241: Harmonische Lage von vier Punkten (Harmonic position of four points).

6. The word, isogonal, might have been preferable, but it is used for another conception (Enzykl. d. Math. Wissensch. as quoted in ref. 3), p. 1218). The expressions, nomogon, equiangular ray, and limiting circle, have also been coined by the author.

7. Die Grundlagen der Brocardschen Geometrie des Dreiecks und die Erweiterung auf das Vieleck (The basis of Brocard's geometry of the triangle and its generalization for the polygon). Jahresbericht d. Mathematiker-Vereinigung. 39. Heft 1/4. Pages 81–88.

8. Instead of the usual way of distinguishing the two equiangular points according to the clockwise or anticlockwise rotation $GA\rightarrow AB$, I have found it more convenient to distinguish them according to the lettering or numbering of the vertices.

9. Mathésis II, 1882, p. 40. Solution to question 68 by E. Lemoine. Point P is defined as the point for which $x^2+y^2-z^2$ is a minimum if x, y, z are the distances from the sides of a triangle. The solution is $x:-a=y:-b=z:c$.

10. Mathésis V, 1885. Lemoine, Propriétés diverses du cercle et de la droite de Brocard.

11. A very elegant construction of Moebius' quadrangle and accordingly of the symmedian has been devised by E. Eckhardt (Zeitschrift für mathematischen und naturwissenschaftlichen Unterricht 36, p. 417).

THE WILLIAM LOWELL PUTNAM MATHEMATICAL COMPETITION

G. W. MACKEY, Harvard University

The following results of the eighth annual William Lowell Putnam Mathematical Competition held March 20, 1948, have been determined in accordance with the rules of the Competition agreed to by the representatives of the Mathematical Association and the trustees of the William Lowell Putnam Intercollegiate Memorial Fund. The contestants were known to the grader only by number.

The first prize, four hundred dollars, is awarded to the Department of Mathematics of Brooklyn College, Brooklyn, New York. The members of the team were L. Geller, M. Hausner, S. Schwartzman; to each of these a prize of forty dollars is awarded.

The second prize, three hundred dollars, is awarded to the Department of Mathematics of the University of Toronto, Toronto, Ontario, Canada. The members of the team were G. F. D. Duff, J. M. Kennedy, I. A. Rosenberg. To each of these a prize of thirty dollars is awarded.

The third prize, two hundred dollars, is awarded to the Department of Mathematics of Harvard University, Cambridge, Massachusetts. The members of the team were W. Stinespring, D. L. Yarmush, A. Zemach; to each of these a prize of twenty dollars is awarded.

The teams of the Departments of Mathematics of the College of the City of New York and McGill University, Montreal, Quebec, Canada, tied for fourth place. The fourth prize of one-hundred dollars, will be divided equally between the two schools. The members of the team of the College of the City of New York were D. J. Newman, M. Davis, E. Fischer. The members of the team of McGill University were H. Gonshor, B. A. Rattray, and D. D. Patterson. To each of these a prize of five dollars is awarded.

The six* persons ranking highest in the examination, named in alphabetical order, were: G. F. D. Duff, University of Toronto; L. Geller, Brooklyn College; H. Gonshor, McGill University; R. L. Mills, Columbia University; D. J. Newman, College of the City of New York; E. L. Whitney, University of Alberta. Each of these will receive a prize of forty dollars.

The four succeeding persons ranking highest in the examination, named in alphabetical order, were: J. Ehrman, University of Pennsylvania; M. Gerstenhaber, Yale University; M. Hausner, Brooklyn College; W. F. Stinespring, Harvard University. To each of these a prize of twenty dollars is awarded.

The following teams, named in alphabetical order, won honorable mention: University of Alberta, Edmonton, Alberta, Canada, the members of the team being I. A. Lesk, Mrs. L. Parks, E. L. Whitney; Carnegie Institute of Technology, Pittsburgh, Pennsylvania, the members of the team being J. F. Nash, D. L.

* Here two men tied for fifth place; hence the first six instead of the usual first five.

Wallace, H. F. Weinberger; Columbia University, New York, New York, the members of the team being R. L. Feldmann, C. F. Langley, R. L. Mills; University of Pennsylvania, Philadelphia, Pennsylvania, the members of the team being J. Ehrman, M. E. Hamstrom (Miss), and W. J. Turanski; Yale University, New Haven, Connecticut, the members of the team being L. Carteret, M. Gellmann, and M. Gerstenhaber.

Eleven individuals were given honorable mention. The names are listed in alphabetical order. M. Davis, College of the City of New York; M. Djorup (Miss), Ursinus College; M. Gutterman, College of the City of New York; J. F. Nash, Carnegie Institute of Technology; J. B. Patterson, University of Toronto; B. A. Rattray, McGill University; I. A. Rosenberg, University of Toronto; S. Schwartzman, Brooklyn College; R. Spinelli, College of the City of New York; H. F. Weinberger, Carnegie Institute of Technology; D. L. Yarmush, Harvard University.

The following is a list of all colleges and universities which entered teams in the competition. The list, in alphabetical order, is: University of Alberta, University of British Columbia, Brooklyn College, Carleton College, Carnegie Institute of Technology, City College of the College of the City of New York, College of St. Thomas, Columbia University, Harvard University, Howard University, Macalester College, Massachusetts Institute of Technology, McGill University, University of Michigan, University of Oregon, University of Pennsylvania, Queen's College (Flushing, N. Y.), Queen's University (Kingston, Ontario), Swarthmore College, Texas Technological College, University of Toronto, Ursinus College, United States Naval Academy, and Yale University.

The following additional colleges and universities entered individual contestants only: Holy Cross College, Loyola College (Montreal), Oklahoma Agricultural and Mechanical College, Rutgers University, and the University of Saskatchewan.

A total of 120 undergraduates representing 29 institutions took part in the competition.

Participants in the competition were given the following lists of problems.

Part I. Three Hours

(Answer the questions in any order and by any method. Show all your work in logical sequence, and indicate your answers clearly. No tables or other books may be used.)

1. What is the maximum of $|z^3-z+2|$, where z is a complex number with $|z|=1$?

2. Two spheres in contact have a common tangent cone. These three surfaces divide the space into various parts only one of which is bounded by all three surfaces; it is "ring-shaped." Being given the radii of the spheres, r and R, find the volume of the "ring-shaped" part. (The desired expression is a rational function of r and R.)

3. Let $\{a_n\}$ be a decreasing sequence of positive numbers with limit 0 such that

$$b_n = a_n - 2a_{n+1} + a_{n+2} \geqq 0$$

for all n. Prove that

$$\sum_{n=1}^{\infty} nb_n = a_1.$$

4. Let D be a plane region bounded by a circle of radius r. Let (x, y) be a point of D and consider a circle of radius δ and center at (x, y). Denote by $l(x, y)$ the length of that arc of the circle which is outside D. Find

$$\lim_{\delta \to 0} \frac{1}{\delta^2} \int\int_D l(x, y)dxdy.$$

5. If $x_1, \cdots, x_n$ denote the n-th roots of unity, evaluate

$$\pi(x_i - x_j)^2 \qquad (i < j).$$

6. Answer either (a) or (b):

(a) A force acts on the element ds of a closed plane curve. The magnitude of this force is $r^{-1}ds$ where r is the radius of curvature at the point considered, and the direction of the force is perpendicular to the curve; it points to the convex side. Show that the system of such forces acting on all elements of the curve keep it in equilibrium.

(b) Show that

$$x + \frac{2}{3}x^3 + \frac{2}{3}\frac{4}{5}x^5 + \frac{2}{3}\frac{4}{5}\frac{6}{7}x^7 + \cdots = \frac{\arcsin x}{\sqrt{1-x^2}}.$$

Part II. Three Hours

(Answer the questions in any order and by any method. Show all your work in logical sequence, and indicate your answers clearly. No tables or other books may be used.)

1. Let $f(x)$ be a cubic polynomial with roots x_1, x_2 and x_3. Assume that $f(2x)$ is divisible by $f'(x)$ and compute the ratios $x_1 : x_2 : x_3$.

2. "A penny in a corner." A circle moves so that it is continually in contact with all three coordinate planes of an ordinary rectangular system. Find the locus of the center of the circle.

3. If n is a positive integer, prove that

$$[\sqrt{n} + \sqrt{n+1}] = [\sqrt{4n+2}],$$

where $[x]$ denotes as usual the greatest integer not exceeding x.

4. Let $\min(x, y)$ denote the smaller of the numbers x and y. For what λ's does the equation

$$\int_0^1 \min(x, y)f(y)dy = \lambda f(x)$$

have continuous solutions which do not vanish identically in (0, 1)? What are these solutions?

5. The pairs of numbers (a, b) such that $|a+bt+t^2| \leqq 1$ for $0 \leqq t \leqq 1$, fill a certain region in the (a, b)-plane. What is the area of this region?

6. Answer *either* (a) or (b):

(a) Let V_1, V_2, V_3 and V denote four vertices of a cube. V_1, V_2 and V_3 are next neighbors of V, that is, the lines VV_1, VV_2 and VV_3 are edges of the cube. Project the cube orthogonally onto a plane (the z-plane, the Gaussian plane) of which the points are marked with complex numbers. Let the projection of V fall in the origin and the projections of V_1, V_2 and V_3 in points marked with the complex numbers z_1, z_2 and z_3, respectively. Show that $z_1^2+z_2^2+z_3^2=0$.

(b) Let a_{ij} be a determinant in which each diagonal element exceeds in absolute value the sum of the absolute values of the other elements of its row, that is $|a_{ii}| > |a_{i1}| + |a_{i2}| + \cdots + |a_{i,i-1}| + |a_{i,i+1}| + \cdots + |a_{in}|$. Show that the determinant is not equal to zero. (Consider the corresponding system of linear homogeneous equations.)

MATHEMATICAL NOTES

EDITED BY E. F. BECKENBACH, University of California
and Institute for Numerical Analysis

Material for this department should be sent directly to E. F. Beckenbach, University of California, Los Angeles 24, California.

THREE CUBIC LOCI

F. C. GENTRY, Arizona State College

The points of contact of the inscribed circle with the sides of a triangle are the vertices of the pedal triangle of the incenter. If these points are joined respectively to the corresponding vertices of the given triangle, the lines thus obtained are concurrent in the Gergonne point of the triangle. The lines joining the corresponding vertices of a triangle and the pedal triangle of its circumcenter are concurrent in the median point. We seek the locus of a point y such that the lines joining the vertices of its pedal triangle to the corresponding vertices of the given triangle may be concurrent in a point z, and the locus of z.

Let A_i, a_i and α_i $(i=1, 2, 3)$, be respectively the vertices, the lengths of the sides, and the angles of a general triangle. Assume a normal trilinear system of coordinates with $A_1A_2A_3$ as reference triangle.

The pedal triangle of $y:(y_1, y_2, y_3)$ has the vertices [1]:

$$P_i:(0,\ y_j + y_i \cos \alpha_k,\ y_k + y_i \cos \alpha_j) \qquad (i, j, k = 1, 2, 3;\ i \neq j \neq k).$$

The lines joining these points respectively to the vertices $A_i:(1, 0, 0)$ are concurrent if the coordinates of y satisfy the equation

$$\sum y_i(y_j^2 - y_k^2)(\cos \alpha_i - \cos \alpha_j \cos \alpha_k) = 0. \tag{1}$$

This is the well-known cubic of Darboux [2]. It is a member of the anallagmatic pencil of cubics obtained by requiring that a point P and its isogonal conjugate with respect to the triangle be collinear with a variable point of the Euler line [3].

The cevian through A_i and z meets A_jA_k in $Q_i:(0, z_j, z_k)$. Perpendiculars to the sides of the triangle at Q_1, Q_2 and Q_3 are concurrent if the coordinates of z satisfy the equation:

$$\sum a_i^2 z_i^2 (z_j \cos \alpha_j - z_k \cos \alpha_k) = 0. \tag{2}$$

This is the cubic of Lucas [2].

The coordinates of z are given in terms of those of y by the transformation:

$$\begin{aligned} \rho z_1 &= (y_3 + y_1 \cos \alpha_2)(y_1 + y_2 \cos \alpha_3) \\ \rho z_2 &= (y_2 + y_1 \cos \alpha_3)(y_3 + y_2 \cos \alpha_1) \\ \rho z_3 &= (y_3 + y_2 \cos \alpha_1)(y_3 + y_1 \cos \alpha_2). \end{aligned} \tag{3}$$

This is essentially a quadratic Cremona transformation of the plane into itself. Its fundamental points are the points at infinity on the altitudes of the triangle through A_1 and A_2 and the symmetric of A_3 as to the circumcenter O. Since all of these points lie on the curve (1), each of the principal curves, the lines joining them in pairs, meets the cubic in one further point, which is therefore the analogue of the corresponding fundamental point of the inverse transformation:

$$\begin{aligned} \rho' y_1 &= z_1 z_3 - z_2(z_3 \cos \alpha_3 - z_1 \cos \alpha_1) \\ \rho' y_2 &= z_2 z_3 - z_1(z_3 \cos \alpha_3 - z_2 \cos \alpha_2) \\ \rho' y_3 &= z_3^2 - (z_1 \cos \alpha_1 - z_3 \cos \alpha_3)(z_2 \cos \alpha_2 - z_3 \cos \alpha_3). \end{aligned} \tag{4}$$

The fundamental points of (4) are the vertices A_1 and A_2 and the ex-median point opposite A_3. They also lie on the curve (2) so that the principal curves of (4) meet the cubic in one variable point each, which therefore corresponds to the fundamental point of (3). There is therefore a one-to-one correspondence, without exception, between the points of (1) and the points of (2).

The lines joining the points of contact of the excircles, and the sides to

which they are relative, to the opposite vertices of the given triangle are concurrent in the Nagel point N. The points of contact are the feet of perpendiculars dropped from the excenters on the corresponding sides of the triangle, which are also known to be concurrent. The excenters, themselves, are the so-called associated points of the incenter. In a manner similar to that outlined above, it may easily be shown that if a point y traces the 17-point cubic:

$$\sum a_i y_i^2(a_j y_k - a_k y_j) = 0, \tag{5}$$

then the perpendiculars let fall from the associated points of y on the corresponding sides of the triangle are concurrent in a point of the cubic of Darboux (1) and the lines joining the feet of these perpendiculars to the opposite vertices of the triangle are concurrent in a point of the cubic of Lucas (2). The 17-point cubic is a member of the anallagmatic pencil mentioned above [4].

In each case the transformation and its inverse are quadratic. The fundamental points lie on the curve concerned so that a one-to-one correspondence is set up between the two curves.

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1. F. C. Gentry, Analytic Geometry of the Triangle, National Mathematics Magazine, vol. 16, 1941, p. 129.
2. R. Deaux, Sur les Cubiques de Darboux et de Lucas, Mathesis, vol. 38, 1924, p. 430.
3. J. R. Musselman, Some Loci Connected with a Triangle, this MONTHLY, vol. 47, 1940, p. 360.
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ANALLAGMATIC CUBICS

J. M. FELD, Queens College, New York

In a paper which appeared recently in this MONTHLY [4], C. E. Noble discussed some properties of two cubic curves which remain invariant under the isogonal transformation

$$x_1' : x_2' : x_3' = x_2x_3 : x_3x_1 : x_1x_2, \tag{1}$$

where the x_i are trilinear coordinates of a point in the plane. The cubics discussed by Noble have the equations

$$\alpha_1x_1(x_2^2 - x_3^2) + \alpha_2x_2(x_3^2 - x_1^2) + \alpha_3x_3(x_1^2 - x_2^2) = 0 \tag{A}$$

and

$$\alpha_1x_1(x_2^2 + x_3^2) + \alpha_2x_2(x_3^2 + x_1^2) + \alpha_3x_3(x_1^2 + x_2^2) + \alpha_4x_1x_2x_3 = 0. \tag{B}$$

Although these cubics have received considerable attention in connection with the geometry of the triangle (many references are given by Goormaghtigh [3] and Patterson [6]), it appears to be generally overlooked that all non-singular cubics can, by a suitable choice of the fundamental triangle, be reduced both

to form (A) and to form (B). Singular cubics can be reduced to form (B).

It was proved by Ogura [5] that a cubic C_3 invariant under (1) either passes once through each vertex of the fundamental triangle or else it passes twice through one vertex and once through another. In the former case the equation of C_3 assumes form (A) or form (B). Ogura's results remain valid when the x_i are assumed to be projective coordinates rather than trilinear. The present writer [1, 2] then showed that it is possible, in an infinite number of ways, to choose the fundamental triangle so that the equation of any non-singular C_3 takes either form (A) or form (B), and that therefore every non-singular C_3 is an anallagmatic cubic with respect to infinitely many triangles. Singular cubics, since they can be reduced to form (B), are, of course, also anallagmatic with respect to suitably chosen fundamental triangles.

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3. R. Goormaghtigh, On McCay's cubic, this MONTHLY, vol. 43 (1936), pp. 92–93.

4. C. E. Noble, An anallagmatic cubic, this MONTHLY, vol. 55 (1948), pp. 7–14.

5. K. Ogura, Invariant cubics for isogonal transformations in the geometry of the triangle, Tôhoku Math. Journ., vol. 4 (1913), p. 132.

6. B. C. Patterson, Sur un faisceau de courbes relatives au triangle, L'Enseignement Mathématique, vol. 29 (1930), pp. 279–289.

ON ANALLAGMATIC CUBICS

R. GOORMAGHTIGH, Bruges, Belgium

In his article on anallagmatic cubics (this MONTHLY, vol. 55, 1948, p. 7) C. E. Noble has given several interesting geometric interpretations of the cubic

$$\sum LX(Y^2 + Z^2) + KXYZ = 0$$

in trilinear coordinates.

A remarkable interpretation of the kind has been mentioned by Bouvaist in MATHESIS, 1927, p. 211: *the locus of a point whose pedal circle as to a triangle has a constant power as to a given point is an anallagmatic cubic of the considered type.*

When the cubic splits into a straight line and its isogonal conic, we find the well known theorem of Lemoyne, according to which the pedal circles of the points of a straight line as to a triangle have a constant power as to a fixed point, the orthopole of that line with respect to the triangle.

Bouvaist's paper also contains, p. 214, the geometric interpretation of the cubic as locus of the foci of inscribed conics tangent to a given straight line; this is Theorem 4 in Noble's article and also Theorem 6, as the inscribed conics tangent to a straight line have their centers on another straight line.

ON THE ALTITUDES OF THE TRIANGLE AND OF THE TETRAHEDRON

V. THÉBAULT, Tennie, Sarthe, France

1. Let A_1, B_1, C_1 be the points which divide the altitudes AA', BB', CC' of a triangle $T \equiv ABC$ in the same ratio

$$A_1A : A_1A' = B_1B : B_1B' = C_1C : C_1C' = m : n.$$

Consider masses ka^2, kb^2, kc^2 proportional to the squares a^2, b^2, c^2 of the sides BC, CA, AB, applied at A, B, C and A_1, B_1, C_1. The centroid of these six masses coincides with that of the masses

$$k(m+n)a^2, \qquad k(m+n)b^2, \qquad k(m+n)c^2 \tag{1}$$

applied at the feet A', B', C' of the altitudes of T. But the centroid of masses ka^2, kb^2, kc^2 applied at A, B, C and that of the masses (1) applied at A', B', C' coincide with the Lemoine (symmedian) point K of T [1]. The same holds for the centroid of masses ka^2, kb^2, kc^2 applied at A_1, B_1, C_1. If A_1, B_1, C_1 are on a line Δ, then K is on Δ [2]. There are two lines Δ_1, Δ_2 which divide the altitudes of triangle T in the same ratio. These lines Δ_1, Δ_2 intersect at the Lemoine point K of triangle T [3].

2. Let the points A_1, B_1, C_1, D_1 divide the altitudes AA', BB', CC', DD' of an arbitrary tetrahedron $T \equiv ABCD$ in the same ratio

$$A_1A : A_1A' = B_1B : B_1B' = C_1C : C_1C' = D_1D : D_1D' = m : n.$$

If we apply at the vertices A, B, C, D, and at the points A_1, B_1, C_1, D_1, masses $k\mathfrak{A}^2$, $k\mathfrak{B}^2$, $k\mathfrak{C}^2$, $k\mathfrak{D}^2$ proportional to the squares of the areas of the faces BCD, CDA, DAB, ABC, the centroid of these eight masses coincides with that of the masses

$$k(m+n)\mathfrak{A}^2, \qquad k(m+n)\mathfrak{B}^2, \qquad k(m+n)\mathfrak{C}^2, \qquad k(m+n)\mathfrak{D}^2$$

applied at the feet A', B', C', D' of the altitudes of T. On the other hand, we know that the centroid of masses $k\mathfrak{A}^2$, $k\mathfrak{B}^2$, $k\mathfrak{C}^2$, $k\mathfrak{D}^2$ located at A, B, C, D coincides with the point such that the sum of the squares of its distances from the planes of the opposite faces is a minimum (first Lemoine point K of T) [4]. Furthermore, we have shown that the centroid of masses proportional to the squares of the areas of the faces applied at the feet of the altitudes of a tetrahedron coincides also with the point K [5]. The centroid of the masses $k\mathfrak{A}^2$, $k\mathfrak{B}^2$, $k\mathfrak{C}^2$, $k\mathfrak{D}^2$ applied at A_1, B_1, C_1, D_1 coincides therefore with the first Lemoine point K of T.

If the points A_1, B_1, C_1, D_1 are in the same plane (P), then K is on (P). There are three planes (P_1), (P_2), (P_3) which divide the altitudes of an arbitrary tetrahedron in the same ratio [6]. We have there the following result.

THEOREM. *The three planes* (P_1), (P_2), (P_3) *which divide the altitudes of an arbitrary tetrahedron in the same ratio meet at the first Lemoine point of the tetrahedron.*

Hence in this respect there is complete analogy between the triangle and the tetrahedron.

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E.F.B.

CLASSROOM NOTES

EDITED BY C. B. ALLENDOERFER, Haverford College and Institute for Advanced Study

All material for this department should be sent to C. B. Allendoerfer, Institute for Advanced Study, Princeton, New Jersey.

A REMARK ON INVOLUTORY FUNCTIONS

JOHN ACZÉL, Budapest, Hungary

We say that a function is involutory if it is identical with its inverse function; that is if $y=f(x)$ implies $x=f(y)$. For example $y=a/x$; $y=x$; $y=c-x$; $y=x/(x-1)$ are involutory functions. The graphs of the involutory functions are obviously symmetric with respect to the straight line $y=x$. Therefore we can produce the most general involutory function by drawing an arbitrary curve

in one half of the plane divided by the straight line $y=x$ and by reflecting our curve about this straight line into the other half plane. In the remarks to follow I shall give three analytic ways each of which enables us to construct the most general involutory function.

I. Let $\phi(x, y)$ be an arbitrary symmetric function. Then every solution $y=f(x)$ of the equation $\phi(x, y)=0$ is evidently an involutory function. If $y=f(x)$ is multi-valued, it is necessary to consider all branches of the function, for a single branch may not be involutory. For example, if $\phi(x, y)=\sin x+\sin y$, the particular solution $y=x+\pi$ is not involutory, but the solution $y=x\pm n\pi$ is involutory. Similar remarks apply to the solution of equations given in II and III below. Not only does this procedure give an involutory function, but all involutory functions can be constructed in this fashion. For if $y=f(x)$ is involutory, then $x=f(y)$; and $f(x)+f(y)-(x+y)=0$; and $\phi(x, y)=f(x)+f(y)-(x+y)$ is symmetric. For example the involutory function $y=x/(x-1)$ is a solution of the equation $x/(x-1)+y/(y-1)-(x+y)=0$. It is also, of course, the solution of other symmetric equations such as: $x+y-xy=0$.

In a similar fashion let $\phi(x, y)$ be antisymmetric in x and y. Then any solution of $\phi(x, y)=0$ is involutory, and any involutory function $y=f(x)$ satisfies the equation $\phi(x, y)=f(x)-f(y)-(x-y)=0$.

II. Let $F(x, y)$ be an arbitrary function of two variables. Then $F(x, y)+F(y, x)=0$ is a symmetric function and $F(x, y)-F(y, x)$ is antisymmetric. From I it follows that any solution of $F(x, y)+F(y, x)=0$ or of $F(x, y)-F(y, x)=0$ is involutory, and that any involutory function satisfies both of these relations if we put $F(x, y)=y-f(x)$.

III. Let $\phi(z)$ be any function of a single variable. Then $\phi(x)+\phi(y)$ is a symmetric function and $\phi(x)-\phi(y)$ is antisymmetric. From I it follows that any solution of $\phi(x)+\phi(y)=0$ or of $\phi(x)-\phi(y)=0$ is involutory, and that any involutory function satisfies both of these equations if we put $\phi(x)=x+f(x)$ in the first case and if we put $\phi(x)=x-f(x)$ in the second case.

A THEOREM IN ELEMENTARY MATHEMATICS

A. D. WALLACE, Tulane University

The following definition is adopted from a textbook on algebra: An extraneous root of $f(x)=0$ is a number which is not a root but which satisfies an equation derived from $f(x)=0$ by permissible operations.

Our purpose is to the prove the

THEOREM: *Any number not a root of $f(x)=0$ is an extraneous root.*

Proof: The equation $(x-t)f(x)=0$ is derived from $f(x)=0$ by a permissible operation.

We have (following the author of the textbook) failed to define "equation" and "permissible operation." Such definitions are beyond the scope of this paper. Their significance will, however, be apparent to any thoughtful reader.

In closing we express the hope that our theorem will, in some measure, indicate the importance to be attached to the concept of extraneous root.

ELEMENTARY PROBLEMS AND SOLUTIONS

EDITED BY HOWARD EVES, Oregon State College

Send all communications concerning Elementary Problems and Solutions to Howard Eves, Mathematics Department, Oregon State College, Corvallis, Oregon. This department welcomes problems believed to be new, and demanding no tools beyond those ordinarily furnished in the first two years of college mathematics. To facilitate their consideration, solutions should be submitted on separate, signed sheets, within three months after publication of problems.

PROBLEMS FOR SOLUTION

E 841. *Proposed by C. W. Trigg, Los Angeles City College*

(a) Can any sealed rectangular envelope, after a single straight cut, be folded into two congruent tetrahedra? Will the position of the cut affect the size of the tetrahedra?

(b) How should the cut be made to make the total number of folds and unfolds a minimum?

(c) What should be the relative dimensions of the envelope in order that the tetrahedra be regular?

E 842. *Proposed by H. L. Lee, University of Tennessee*

Determine cubic functions $f(x)$ for which $f(x)=0$ and $f'(x)=0$ have rational roots.

E 843. *Proposed by P. D. Thomas, Washington, D. C.*

Show that the difference in the radii of the two spheres, one equivalent in area, the other equivalent in volume, to an oblate ellipsoid of revolution is of the fourth order in the eccentricity of the generating ellipse.

SOLUTIONS

A Dart Game

E 811 [1948, 248]. *Proposed by H. D. Larson, Albion College, Michigan*

A, B, and C participate in a novel dart game, the targets consisting of three small balloons marked A, B, and C, respectively. At each turn one dart is thrown, the order of the turns being determined in advance by drawing lots. As soon as a balloon is hit and destroyed, the owner of that balloon is eliminated from the game. The balloons are placed in such a manner that there is no danger of destroying a balloon by a dart aimed at another balloon. It is known by all participants that A can hit a balloon 4 out of 5 times, B 2 out of 5 times, and C 2 out of 5 times; this knowledge is used by each player to his best advantage. What is each contestant's chance of winning the game?

Solution by Leo Moser, University of Manitoba. In a two man game, say E *vs.* F, with chance of hit on aim p_1 and p_2 respectively, the probability of E winning if he has first shot is

$$\sum_{i=0}^{\infty} p_1[(1-p_1)(1-p_2)]^i = p_1/[1-(1-p_1)(1-p_2)].$$

It is clear that when all three men are in the game the best strategy for A and B is to try to eliminate each other while C does his best to miss. Thus there are only two essentially different orders of firing, A, B, C and B, A, C, and these have equal probability.

For A to win he must first win a two man game with B and then a two man game with C in which C has first shot. The case for B is similar. Taking these facts into account we find that the chances of A, B, and C winning are 1596/4807 891/4807, 2320/4807, respectively. One observes that the poorest shot has the best chance of winning!

Essentially this problem (somewhat different formulation and different probabilities) is given in Kinnaird, *Encyclopedia of Puzzles and Pastimes*, p. 246. However, the solution there given is incorrect. It is quite likely that certain variations of this problem might have application to economics.

Also solved by Karl Itkin, Roger Lessard, Albert Shaw, John Walker and the proposer.

Lessard, Shaw, and Walker proceeded on the assumption that when all three men are in the game the best strategy for B and C is to try to eliminate A while A's best strategy is to try to eliminate B. Under this assumption they found that the chances of A, B, and C winning are approximately 0.266, 0.304, 0.430. C still has the best chance of winning, but his chances are not so great as under the assumption that when all three men are in the game he should do his best to miss.

ADVANCED PROBLEMS AND SOLUTIONS

EDITED BY E. P. STARKE, Rutgers University

Send all communications concerning Advanced Problems and Solutions to E. P. Starke, Rutgers University, New Brunswick, New Jersey. All manuscripts should be typewritten with double spacing and margins at least one inch wide. Problems containing results believed to be new or extensions of old results are especially sought. Proposers of problems should also enclose any solutions or information that will assist the editor. In general, problems in well known textbooks or results in readily accessible sources should not be proposed for this department.

PROBLEMS FOR SOLUTION

4320. *Proposed by V. F. Ivanoff, San Francisco*

Let $f(s, \phi)=0$ be the intrinsic equation of a simple, closed curve; s is the length of arc of the curve, measured from a fixed point P_0 to any point P, and ϕ is the angle between the tangent lines at P and P_0. Find the area enclosed by the curve.

4321. *Proposed by Paul Erdös, Syracuse University*

Let $n_1<n_2<\cdots<n_k<\cdots$ be a sequence of integers such that

$$\lim_{k\to\infty}\frac{n_k}{n_1n_2\cdots n_{k-1}}=\infty.$$

Show that $\sum_{i=1}^{\infty}1/n_i$ is irrational.

4322. *Proposed by Pedro A. Piza, San Juan, Puerto Rico*

If p is a prime, prove that

$$\binom{n}{p}\equiv[n/p]\pmod{p},$$

where $\binom{n}{p}$ and $[n/p]$ have their usual meanings.

Prove also that when $[n/p]$ contains p^s as a factor, then $\binom{n}{p}$ is divisible by p^s.

4323. *Proposed by Victor Thébault, Tennie, Sarthe, France*

Let $ABCD$ be a convex quadrangle circumscribed about a circle with center O, and let A', B', C', D' be the points of tangency of the sides BC, CD, DA, AB. Consider the circles (OA, B), (OA, D) tangent to OA at O and passing, respectively, through the vertices B, D neighboring A. Consider also the analogous circles tangent to OB, OC, OD at O. (1) The pairs of circles (OA, B) and (OD, C), (OB, C) and (OA, D), (OC, D) and (OB, A), (OD, A) and (OC, B) respectively intersect at N on BC, P on CD, Q on DA, M on AB. (2) The quadrangle $MNPQ$ is a parallelogram with center O having its sides parallel to the diagonals of $ABCD$ and its diagonals parallel to those of $A'B'C'D'$. (3) The centers of the circle and of the equilateral hyperbola circumscribing $A'B'C'D'$, and the Miquel point of the complete quadrilateral formed by the sides of the quadrangle $ABCD$, are collinear.

4324. *Proposed by W. O. Pennell, Exeter, New Hampshire*

When $x\geqq 0$ what is the limit as $n\to\infty$ of

$$\phi(n,x)=e^{-x^n}\left[1+\frac{x^n}{\frac{1}{n}+1}+\frac{x^{2n}}{\left(\frac{1}{n}+1\right)\left(\frac{1}{n}+2\right)}+\frac{x^{3n}}{\left(\frac{1}{n}+1\right)\left(\frac{1}{n}+2\right)\left(\frac{1}{n}+3\right)}+\cdots\right]?$$

SOLUTIONS

Tangent Circles

4253. [1947, 286]. *Proposed by G. T. Williams, Cambridge, Massachusetts*

Given two tangent unit circles, C_1 and C_2, and their common external tangent T. A third circle, C_3, is drawn tangent to C_1, C_2 and T; C_4 is then drawn tangent to C_1, C_2, and C_3; and so on, each C_{j+1} being tangent to C_1, C_2, and C_j. Find the total area of the aggregate of circles, C_1, C_2, C_3, $\cdots$.

Solution by J. Certaine, Howard University, Washington, D. C. Let r_k be the radius of C_{k+2} and let b_k $(k=2, 3, \cdots)$ be the distance between the point of contact of the circles C_1 and C_2 and the point of contact of C_{k+1} and C_{k+2}. $b_1=1$ is the distance from the point of contact of the circles C_1 and C_2 to their common tangent. Then from a figure we have

$$(1 + r_k)^2 = (b_k - r_k)^2 + 1 \quad \text{or} \quad r_k = b_k^2/2(b_k + 1).$$

But $b_{k+1}=b_k-2r_k=b_k/(b_k+1)$. It is easily verified by induction that $b_k=1/k$, whence

$$r_k = \frac{1}{2k(k+1)} = \frac{1}{2}\left[\frac{1}{k} - \frac{1}{k+1}\right].$$

Therefore the total area is

$$A = 2\pi + \pi \sum_{k=1}^{\infty} r_k^2.$$

Now $r_k^2=\{1/k^2+1/(k+1)^2-2/k(k+1)\}/4$. From the known series

$$\sum_{k=1}^{\infty} 1/k(k+1) = 1, \qquad \sum_{k=1}^{\infty} 1/k^2 = \pi^2/6,$$

there result finally

$$\sum_{k=1}^{\infty} r_k^2 = \{\pi^2/6 + (\pi^2/6 - 1) - 2\}/4, \qquad A = (\pi^3 + 15\pi)/12.$$

Also solved by Norman Anning, W. G. Brady, G. Y. Cherlin, N. J. Fine, Free Jamison, Ou Li, Leo Moser, G. A. Williams, and the Proposer. See also *A Family of Integers and a Theorem on Circles* (this MONTHLY, 1947, 534–536), by Williams and Browne.

RECENT PUBLICATIONS

EDITED BY H. P. EVANS, University of Wisconsin

All books for review should be sent directly to the editor of this department, American Mathematical Monthly, 531 West 116th Street, New York 27, N. Y. and not to any of the other editors or officers of the Association.

Rings and Ideals (The Eighth Carus Mathematical Monograph). By N. H. McCoy. LaSalle, Illinois, The Open Court Publishing Company, 1948. 12+216 pages. $3.00. (Members of the Mathematical Association of America may purchase one copy at $1.75, order to be placed with the Secretary-Treasurer).

This book is an extremely carefully written exposition of the fundamentals of the theory of commutative rings. The central core of this topic is the arithmetical theory of prime and primary ideals. The author not only gives a simple presentation of Krull's results, but also provides many interesting glimpses into other branches of our knowledge of rings. Let us turn, then, to a more detailed consideration of the contents of his book.

The first four chapters comprise a leisurely introduction and include a discussion of polynomial rings, ideals, homomorphisms and isomorphisms, imbedding of a ring into a ring with unit element, and rings of quotients. Many elementary examples of rings are introduced and used to illustrate the general considerations. (This excellent practice is continued throughout the monograph.) The use of the unexplained word *indeterminate* in the definition of *polynomial* and the introduction of *ideals* prior to *homomorphisms* were the only points which seemed inconsistent with the high level of careful exposition maintained elsewhere in these chapters.

The remainder of the book treats selected topics from the theory of commutative rings. A basis for a theory of prime ideals is deduced in Chapter V by means of the Maximum Principle of Zorn. The discussion turns to structure theory in Chapter VI where a commutative ring is characterized as a subdirect sum of fields if and only if its Jacobson radical is zero. This result is applied to Boolean rings, to rings in which $x^p=x$ and $px=0$ for some prime p and all x, and to the regular rings of von Neumann in Chapter VII. Rings of matrices with elements in a commutative ring with unit are treated in Chapter VIII. This theory is due to the author and concerns the relations between the null ideal and the characteristic ideal of a given matrix. Chapter VIII also contains an economical development of the properties of determinants and resultants in a commutative ring with unit element based on the results of the usual theory for fields. Primary ideals and a generalization of the fundamental theorem of arithmetic to commutative rings which satisfy the ascending chain condition are studied in the last chapter. There is an indication at the close of the importance of this theory of ideals in algebraic geometry. A brief bibliography and an index of terms is appended.

The author has made every effort to lead the reader gradually from the very

definition of a ring to the useful and advanced results of his last chapter. There is no question in this reviewer's mind of the great value of this monograph as a text for graduate students. It is true that no attempt is made to cover a wide variety of topics—only those topics are studied which fall naturally into the logical pattern selected. But this is the virtue of the book, since the student is presented with material which is sufficiently general to be suggestive and *useful* in his own efforts at creative work and which is, at the same time, not so difficult as to require an unreasonable amount of time for assimilation.

If the expository powers of this reviewer could match those of the author, it would now be clear that the Eighth Carus Monograph is a welcome addition to the literature of algebra and is truly a report of some of "the best thoughts and keenest researches" in this field.

M. F. SMILEY

Projective and Analytical Geometry. By J. A. Todd. New York, Pitman Publishing Company, 1948. 10+289 pages. $4.50.

This book was written as a textbook for Part II of the Mathematical Tripos at Cambridge. The treatment throughout is in terms of homogeneous real or complex coordinates. Extensive study is made not only of conics and quadric surfaces but also of the twisted cubic. An elementary knowledge of matrices is presupposed and the chapter on collineations is concerned with reduction of matrices to canonical form, and includes the relevant material on elementary divisors. Since the projective properties of geometric figures are those invariant under the transformations of the projective group, in an elegant and natural way the theory of invariants is introduced and related to geometric properties. For example: Given two homogeneous ternary quadratic forms S and S^1, the condition that there exist triangles inscribed in the conic $S^1=0$ which are self-polar with respect to $S=0$ is given by the vanishing of an invariant of S and S^1.

Almost no synthetic proofs are given. The modern theory of introduction of coordinates and the role of the Theorem of Desargues and the Theorem of Pappus are omitted. Reference is made to modular coordinate fields, but none to non-commutative division rings as coordinates.

MARSHALL HALL, JR.

NEW BOOKS RECEIVED

Ebene und Sphärische Trigonometrie. By H. Athen. (Bücher der Mathematik und Naturwissenschaften.) Wolfenbutteler, Wolfenbuttel and Hannover, 1948. 112 pages.

Analytische Geometrie. By W. Blaschke. (Bücher der Mathematik und Naturwissenschaften.) Wolfenbutteler, Wolfenbuttel and Hannover, 1948. 152 pages.

Vektorrechnung. By H. Athen. (Bücher der Mathematik und Naturwissenschaften.) Wolfenbutteler, Wolfenbuttel and Hannover, 1948. 2+90 pages.

Tables of the Bessel Functions of Fractional Order. Vol. I. Prepared by the Computation Laboratory of the National Bureau of Standards. New York, Columbia University Press, 1948. 42+413 pages. $7.50.

CLUBS AND ALLIED ACTIVITIES

EDITED BY L. F. OLLMANN, Hofstra College

Send reports of all activites, such as club reports, special features, topics with references, student papers, and other material of interest to L. F. Ollmann, Hofstra College, Hempstead, New York.

CLUB REPORTS 1947–48

Mathematics Club, University of Dayton

Biweekly meetings were held at which members presented the following papers:

Bessel functions and their application to frequency modulation, by Daniel Groszewski

A novel trigonometric reduction rule, by J. F. Schell

Dimensional analysis, by P. F. Swift

Atomic models, by H. J. Eichel

Impossibility of trisection of all angles by ruler and compasses, by E. J. Freeh

Hyperbolic functions and their uses, by Thomas Hanlon

Calculus of variation, by C. L. Keller

Mathematical logic, by Frank Levin

Cardan's solution of a cubic equation, by Robert Luthman

Controversy concerning the invention of the calculus, by Thomas Beckert.

The members attended a lecture at the University of Cincinnati given by Professor G. Birkhoff of Harvard University, who spoke on *The effects of small vibrations on incompressible liquids*, and assisted in the Fourth Annual Colloquium of the National Mathematics Honor Society of Secondary Schools at Mother of Mercy High School in Cincinnati, Ohio.

Preceding the annual dinner, Professor R. V. Churchill of the University of Michigan spoke on *Applications of the Laplace transformation*. The Dean of Science Award was conferred upon P. F. Swift for his paper presented during the first semester. The award for the second semester was conferred upon C. L. Keller. The Alumni Calculus Prize to the member of the sophomore class who made the highest achievement in a competitive integral calculus examination was presented to G. T. Ryan.

At commencement the Mathematics Club Alumni Awards of Excellence in advanced mathematics were given to C. L. Keller of the senior class and upon Robert Luthman of the junior class.

Officers during the year were: President, E. J. Freeh; Vice-President, P. F. Swift; Secretary, G. D. Moon; Treasurer, John Quinlisk; Publicity Secretary, Daniel Groszewski; Faculty advisor, Dr. K. C. Schraut.

Mathematics Club, Berea College

The mathematics majors of Berea College, under the supervision of Professors W. R. Hutcherson, G. G. Roberts, D. W. Pugsley and Ruth Porter,

presented on March 20, 1948, a mathematics and astronomy exhibit as a part of the annual Science Open House.

By the use of interesting displays, mechanical devices, graphs and charts, the students planned and demonstrated the following exhibits:

Applications of calculus, by James Howard and Robert Rogers
Mechanical exhibit of derivative, by Robert Lufburrow
Mathematics Club activities, by Sarah Ann Hutcherson and Rosa Lee Case
Statistics, by Nancy Furry, Wanda Hammons, and Delpha Davis
Computing machines, by Betty Shaffer, Nella Walker, and June Puckett
Astronomy and surveying instruments, by Prof. G. G. Roberts and Jack Hale
Conic sections and interesting solids, by Russel Dean and Lee Wickline
Telescope, by Professor D. W. Pugsley
Slide rule, by Charles Hibbits and Andre Rieben
Puzzles, by Jo Crotchfield, Jacky Hopper, and "Doc" Stevens
Concentrations of liquids, by Earl and Ora Lee Skeen
Mechanical drawing, by Ernest Graham and Richard Parker.

As a result of the students' enthusiasm, the Mathematics Club was organized to further promote interest in mathematics and astronomy in the college. At the first meeting the constitution was adopted by the twenty-five charter members.

The officers elected were: President, Lee Wickline; Vice-president, Wanda Hammons; Secretary-treasurer, Earl Skeen; Faculty advisor, Dr. W. R. Hutcherson.

Mathematics Club, Iowa State College

The Mathematics Club of Iowa State College held six regular meetings during the academic year, 1947–48. The purpose of the club, which is sponsored by the *Iowa Alpha* Chapter of *Pi Mu Epsilon*, is to promote interest in mathematics among the freshman and sophomore students of the college. Refreshments were served and the following programs were given at these meetings:

Mathematics in life insurance, by G. A. Harper, vice-president of Bankers Life Insurance Co., Des Moines, Iowa.

Mathematical quiz show, with H. D. Block as master of ceremonies.

Pi, its mathematical uses and origin. Three short papers by J. E. K. Smith, Robert Miller, and William Ritts.

Mathematics in other countries, talks given by student and faculty representatives from India, Russia, Sweden and Poland.

Mechanical aids to computation

Astronomy—A challenge from the heavens, by Prof. Fred Brandner of Iowa State College.

Officers of the club include: Vice-director of Pi Mu Epsilon in charge of the Mathematics Club, Ruth Royer; Program Chairman, Ruth Rychnovsky; Refreshment Committee, Charles Hopper, Norman Burch; Publicity Chairman, James Mairs; Faculty Advisors, Fred Robertson, C. H. Lindahl, R. J. Lambert.

NEWS AND NOTES

EDITED BY EDITH R. SCHNECKENBURGER, University of Buffalo

Readers are invited to contribute to the general interest of this department by sending news items to Edith R. Schneckenburger, University of Buffalo, Buffalo 14, New York. Items should be submitted at least two months before publication can take place.

CONFERENCES AT OHIO STATE UNIVERSITY

Two conferences, sponsored by the Department of Mathematics and the Graduate School of Ohio State University, will be held in Columbus on December 27, 1948. One conference will pertain to surface area theory and one to group theory. Information can be obtained by writing to the Department of Mathematics.

PERSONAL ITEMS

Professor Emil Artin of Princeton University has been appointed to one of the Sigma Xi national lectureships for 1948–49.

Professor Elie Cartan, University of Paris, and Professor Marston Morse, Institute for Advanced Study, have been awarded the honorary degree of Doctor of Science by the University of Pisa.

Dr. H. M. MacNeille, chief of the Fundamental Research Branch, Division of Research, United States Atomic Energy Commission, has been awarded the President's Certificate of Merit in recognition of his services as a member of the resident staff of the London Mission of the Office of Scientific Research and Development from February, 1944 to February, 1946. Other mathematicians who have received the Certificate of Merit for outstanding service to the Government are: Dr. Marston Morse, Dr. A. H. Taub, Dr. Oswald Veblen, Dr. S. S. Wilks.

Professor Tibor Radó has been named a corresponding foreign member of the Academy of Science of Bologna, Italy.

Boston University announces the following: Assistant Professor R. N. Johanson has been promoted to an associate professorship; Mr. J. E. Alman, Dr. J. B. Giever, and Dr. F. J. Scheid have been appointed to instructorships.

At Catholic University of America the following appointments to instructorships have been made: Dr. Paul Nesbeda, formerly at the Institute for Advanced Study, and Mr. R. N. Huckins of Suffolk University.

Colorado Agricultural and Mechanical College announces the promotions of Associate Professor H. T. Guard to a professorship and Assistant Professor M. L. Madison to an associate professorship.

Columbia University announces: Associate Professor J. A. Northcott has been promoted to a professorship; Mr. E. R. Kolchin and Mr. Howard Levi have been promoted to assistant professorships; Mr. J. A. Zilber has been appointed Lecturer; Mr. John Rausen has been appointed Assistant.

Cornell University announces the following: Professor W. H. J. Fuchs of the

University of Liverpool is serving as visiting associate professor for the current academic year; Mr. M. S. Donsker of the University of Minnesota and Dr. Christine Williams of Yale University have been appointed to instructorships; two graduate students, Samuel Goldberg and G. M. Wing, received Atomic Energy Fellowships for 1948–49.

At Florida State University: Associate Professor W. H. Spragens of Georgia State Woman's College has been appointed to an associate professorship; Mr. J. L. Bagg, Western Maryland Teachers College, and Miss Frances Dean, graduate student at University of Alabama, have been appointed to instructorships; Miss Margaret Cockrell, formerly teaching assistant, is now employed as mathematician by the Shell Oil Company.

Hastings College announces: Professor R. M. McDill has become Professor Emeritus; Mr. Everett Lowry, formerly of the Veteran's Administration in Nebraska, has received an appointment in the Mathematics Department.

At Indiana University: Professor V. Hlavaty, formerly of Charles University, Prague, has been appointed Visiting Professor; Professor Eberhard Hopf is employed on the ONR project; Professor D. Gilbarg attended New York University last summer in connection with the ONR project work; Professors W. Gustin and T. Y. Thomas attended California Institute of Technology during the summer in connection with the ONR project work.

Iowa State College makes the following announcements: Assistant Professor Bernard Vinograde has been promoted to an associate professorship; Dr. R. A. Griffin has been appointed to an assistant professorship; Mr. R. D. Branstetter, Mrs. M. L. Butler, Mr. R. F. Deniston, Miss Mary B. Lieberknecht, Miss Ruth Royer, and Mrs. C. C. Scott have been appointed to instructorships. Also, Dr. B. R. Seth of Hindu College, Delhi, India will be Visiting Professor of Applied Mathematics during the year 1949; he will give courses in elasticity and hydrodynamics during the Winter and Spring Quarters.

Kent State University announces: Assistant Professor John Kaiser has been promoted to an associate professorship; Associate Professor Foster Brooks is on leave of absence for the year and is connected with the Research and Development Board of the National Military Establishment, Washington, D. C.

At Iowa State Teachers College the following appointments to instructorships have been made: Miss Dorothy DeWitt and Mr. Robert Lankton.

Massachusetts Institute of Technology announces the following: Associate Professor Witold Hurewicz has been promoted to a professorship; Dr. Walter Pitts has been appointed Research Associate in Mathematics and Dr. R. P. Boas, Jr. has been appointed Lecturer. Professors Warren Ambrose and Norman Levinson are on leave of absence this year on Guggenheim fellowships; Professor Ambrose is spending the year at the Institute for Advanced Study and Professor Levinson is at the University of Copenhagen. Professor Henry Wallman, who is on leave of absence, is Visiting Professor of Teletechnics at the Chalmers Institute of Technology in Gothenburg, Sweden.

Miami University reports: Professor H. S. Pollard, formerly acting head, is now Head of the Department of Mathematics; Dr. R. W. Bryant, graduate student at the University of Alabama, has been appointed to an instructorship; Miss Kathryn B. Aldrich, Mr. R. P. Bacon, Mr. R. H. Johnston and Mr. G. E. Witter are newly appointed graduate assistants; Professor Emeritus W. E. Anderson has retired.

Michigan State College announces the following appointments: Dr. L. M. Kelly of the University of Missouri, Dr. Mary H. Payne of the University of Detroit, and Dr. M. D. Springer of the University of Illinois to assistant professorships; Miss Betty S. Grossman, Miss Roslyn Hurwitz, and Mr. Leo Lapidus to instructorships.

At Michigan State Normal College: Miss Mildred J. Brannon, graduate assistant at the University of Wisconsin, has been appointed to an assistant professorship; Assistant Professor R. G. Peterson is now in charge of visual education.

The following promotions have been made at Mississippi State College: Associate Professor Arthur Ollivier to a professorship; Assistant Professor S. B. Murray to an associate professorship; Miss Monica Goen, Mr. A. C. Grimes, and Mr. M. M. Temple to assistant professorships.

Ohio State University reports: Associate Professors Marshall Hall, H. B. Mann, and P. V. Reichelderfer have been promoted to professorships; Assistant Professors H. H. Alden, R. G. Helsel, and E. J. Mickle have been promoted to associate professorships; Instructor R. R. Whitney has been promoted to an assistant professorship; Dr. O. W. Rechard has been appointed to an instructorship.

Oklahoma Agricultural and Mechanical College makes these announcements: Professor J. C. C. McKinsey has been granted a leave of absence to participate in the Rand Project at Douglas Aircraft, Santa Monica, California; Associate Professor P. E. Lewis has been granted a leave of absence for teaching and advanced study at North Carolina State; Assistant Professor R. R. Reynolds has been granted a leave of absence for advanced study at Harvard University; Dr. Nachman Aronsazjn has been appointed Visiting Professor for the period October 18 to December 18; Dr. M. L. Richards has been appointed Instructor and Research Assistant.

Rutgers University announces the following promotions: Instructors Erwin Biser, L. M. Court, and L. M. Rauch to assistant professorships.

Texas State College for Women reports: Professor Harlan C. Miller has been appointed Director of the Department of Mathematics; Miss Jean Beal, formerly graduate assistant at Oklahoma Agricultural and Mechanical College, has been appointed to an instructorship; Dean F. V. White has retired with the title of Professor Emeritus.

The University of California at Los Angeles announces the following promotions and appointments: Associate Professor P. G. Hoel has been promoted

to a professorship; Assistant Professors J. W. Green and W. T. Puckett have been promoted to associate professorships; Instructors Leonard Greenstone and Alfred Horn have been promoted to assistant professorships; Dr. Robert Steinberg, University of Toronto, and Dr. E. G. Straus, formerly of the Institute for Advanced Study, have been appointed to instructorships.

At the University of Delaware: Associate Professor G. C. Webber has been promoted to a professorship; Dr. R. F. Jackson, formerly a research mathematician with York Refrigerating Company, York, Pennsylvania, has been appointed Assistant Professor; Mr. Morris Neuman of the University of Pennsylvania has been appointed Instructor.

The University of Denver announces the promotions of Professor A. J. Lewis to the Chairmanship of the Department of Mathematics and Assistant Professor K. L. Noble to the associate chairmanship.

The University of Detroit makes the following announcements: Associate Professor L. E. Mehlenbacher has been made Professor and Director of the Department of Mathematics; Professor Jerzy Lubelfeld of the Polish Naval College and Polish Technical Training Center, Okehampton, Devon, England has been appointed Assistant Professor: Assistant Professor Emily C. Pixley of Wayne University has been appointed to an assistant professorship; Assistant Professor G. E. Markle has been granted a leave of absence for graduate study at Harvard University.

Dr. H. A. Arnold has been appointed to an instructorship at the University of California at Davis.

Miss Laura Blakeley, who has been teaching at Dayton Senior High School, Kentucky, is now an instructor at Armstrong Junior College.

Miss Margaret Bootz has been appointed to an instructorship at Marietta College.

Associate Professor H. W. Brinkmann of Swarthmore College has been promoted to a professorship.

Mr. Paul Brock of Hunter College has accepted a position as mathematician with the Reeves Instrument Company of New York City.

Associate Professor D. M. Brown of Central Michigan College has been appointed Supervisor of the Applied Mathematics Group, Aeronautical Research Center, University of Michigan.

Professor Emeritus W. D. Cairns of Oberlin College is serving as part-time lecturer in mathematics at California Institute of Technology for the year 1948–49.

Assistant Professor R. H. Cole of the University of Western Ontario has been appointed Research Associate at Princeton University.

Mr. R. A. Deutsch of the Triborough Bridge and Tunnel Authority has accepted a position as electrical engineer with the Kellex Corporation of New York City.

Professor Alexander Dillingham of the United States Naval Academy has retired.

Dr. Mary P. Dolciani, who has been at the Institute for Advanced Study, has been appointed to an instructorship at Vassar College.

Mr. E. J. Downie of Colgate University has been promoted to an assistant professorship.

Mr. R. P. Eddy has joined the Applied Mathematics Section of the Mechanics Division, Naval Ordnance Laboratory, Silver Spring, Maryland.

Assistant Professor A. B. Farnell of the University of Colorado has been appointed Lecturer at Princeton University.

Miss Virginia I. Felder of Odessa Junior College has been appointed to an assistant professorship at Mississippi Southern College.

Assistant Professor Landis Gephart of the University of Dayton has accepted a position as mathematician with the United States Air Force.

Mr. M. T. Goodrich, Keene Teachers College, New Hampshire, has been promoted to an assistant professorship.

Mr. Arthur Grad has accepted an appointment as mathematician with the Mathematics Branch of the Office of Naval Research at Washington, D. C.

Mr. G. B. Haggerty of Rhode Island State College has been promoted to an assistant professorship.

Associate Professor F. F. Helton of Central College has been promoted to a professorship.

Assistant Professor Manuel Herschdorfer, Seton Hall College, has been promoted to a professorship.

Mr. R. J. Howerton of the University of Denver has been appointed Instructor in Mathematics and Physics at Regis College.

Associate Professor S. A. Jennings of the University of British Columbia has been promoted to a professorship

Dr. C. M. Jensen, University of Minnesota, has been appointed to an assistant professorship at Augustana College.

Professor B. W. Jones of Cornell University has accepted a professorship at the University of Colorado.

Mr. H. P. Kean has been appointed to an associate professorship at McMurry College.

Assistant Professor Fred Kiokemeister, Mount Holyoke College, has been promoted to an associate professorship.

Professor W. I. Layton of Austin Peay State College has accepted an appointment as associate professor at Alabama Polytechnic Institute.

Assistant Professor A. T. Lonseth of Northwestern University has been appointed to an associate professorship at Oregon State College.

Assistant Professor Kenneth May, Carleton College, has been promoted to an associate professorship.

Assistant Professor P. E. Meadows of Washington and Lee University has been appointed to an associate professorship at Carroll College.

Professor L. I. Mishoe of Delaware State College has been appointed to an associate professorship of physics at Morgan State College.

Associate Professor E. W. Montroll of the University of Pittsburgh has been promoted to a professorship.

Mr. M. R. Moore has been appointed to an instructorship at Bowling Green State University.

Dr. F. M. Morgan, former headmaster of Clark School, Hanover, New Hampshire, has been appointed Treasurer of Clark School Foundation.

Dr. E. N. Nilson, who has been serving as an aeronautical engineer, Research Division, United Aircraft Corporation, has been appointed to an assistant professorship at Trinity College.

Assistant Professor S. T. Parker of Kansas State College has been promoted to an associate professorship.

Professor A. W. Philips of Kansas State Teachers College has been appointed to a professorship at Eastern Washington College of Education.

Assistant Professor D. H. Rock of Iowa State College is now engaged in research work with the United States Army Air Force, Washington, D. C.

Mr. C. T. Rodney of Sampson College has been appointed Assistant Professor at New York State Teachers College, Buffalo.

Dr. R. H. Stark of Northwestern University is now employed at Los Alamos Scientific Laboratory, New Mexico.

Assistant Professor J. C. Stewart, Lawrence College, has been promoted to an associate professorship.

Assistant Professor Feodor Theilheimer of Trinity College has joined the Naval Ordnance Research Laboratory.

Dr. D. L. Thomsen, Jr. of Haverford College has been promoted to an assistant professorship.

Assistant Professor Eleanor B. Walters, Delta State Teachers College, has been promoted to an associate professorship.

Associate Professor C. P. Wells, who is on leave of absence from Michigan State College, is Lecturer at Brown University.

Assistant Professor G. N. Wollan of Sampson College has been appointed to an associate professorship at North Georgia College.

Dr. Marie A. Wurster of Temple University has been promoted to an assistant professorship.

Dr. Walter Mayer, associate in Mathematics at the Institute for Advanced Study, died on September 10, 1948.

Professor W. A. Wilson of Yale University died on September 21, 1948. He was a charter member of the Association.

THE MATHEMATICAL ASSOCIATION OF AMERICA

Official Reports and Communications

THE NINTH ANNUAL WILLIAM LOWELL PUTNAM MATHEMATICAL COMPETITION

Professor L. E. Bush of the College of St. Thomas, St. Paul 1, Minnesota, has been appointed Director of the William Lowell Putnam Mathematical Competition by President L. R. Ford for the five-year period 1949–1953. At the recent meeting of the Board of Governors in Madison, Wisconsin, it was voted that the Association should resume direction of the Putnam Competition and the appointment of a Director was authorized. This competition, made possible by the trustees of the William Lowell Putnam Intercollegiate Memorial Fund left by Mrs. Putnam in memory of her husband, is open to undergraduate students in universities and colleges of the United States and Canada who have not received a degree.

The ninth annual Putnam Competition will be held on Saturday, March 26, 1949. The examination will consist of two parts of three hours each. The questions will be taken from the fields of calculus (elementary and advanced) with applications to geometry and mechanics not involving techniques beyond the usual applications, higher algebra (determinants and theory of equations), elementary differential equations, and geometry (advanced plane and solid analytic geometry). Any college or university wishing to enter a team or individual contestants may secure an application blank from Professor L. E. Bush, 112 Albertus Magnus Hall, College of St. Thomas, St. Paul 1, Minnesota, by a postcard request. All applications must be filed with the Director not later than March 1, 1949. If three candidates are presented from a college or university, they are to constitute a team; if more than three are presented from any one college or university, the team of three must be named on the application. Fewer than three from one college or university may compete as individuals.

The examination may be given at any place where a team, or at least three candidates, can be assembled. Exceptions to this rule may be made by the Director in cases where it would entail unusual inconvenience to a contestant. Sealed copies of the examinations will be sent to the supervisor of the examination in time for the examination day and are not to be opened before the hour set. At the supervisor's first opportunity after the afternoon examination the books are to be sent by registered mail or by express to the Director, who will forward them to a qualified reader chosen by the Association.

The prizes to be awarded to the departments of mathematics of the institutions with the winning teams are \$400, \$300, \$200, and \$100, in the order of their rank. In addition, there will be prizes of \$40, \$30, \$20 and \$10 awarded to the members of these teams according to the rank of the team; a prize of \$50 to each of the five highest contestants and a prize of \$20 to each of the succeeding five highest contestants. Each of the winners will receive a suitable medal. Honorable mention will be given to several teams next in order after the four

winning teams and to several individuals next in order after the ten individual winners. For further encouragement of the Competition, there will be awarded at Harvard University (or at Radcliffe College in the case of a woman) an annual $1500 William Lowell Putnam Prize Scholarship to one of the first five contestants, this to be available either immediately or on the completion of the student's undergraduate work.

Reports on the eight previous competitions and examination questions will be found in this MONTHLY for May, 1938, 1939, 1940, 1941, 1942, October, 1946, August–September, 1947, and in this issue.

APRIL MEETING OF THE IOWA SECTION

The thirty-fifth annual meeting of the Iowa Section of the Mathematical Association of America was held at Parsons College, Fairfield, Iowa, on Friday and Saturday, April 16 and 17, 1948, in conjunction with the Iowa Academy of Science. Professor H. P. Thielman, Chairman of the Section, presided.

The attendance was seventy, including the following twenty-nine members of the Association: E. W. Anderson, E. L. Canfield, E. W. Chittenden, Marian E. Daniells, W. M. Davis, R. E. Gaskell, B. E. Gillam, Cornelius Gouwens, F. S. Harper, Gertrude A. Herr, E. H. C. Hildebrandt, J. J. L. Hinrichsen, L. A. Knowler, O. C. Kreider, C. E. Langenhop, Walter Lyche, R. B. McClenon, F. M. McGaw, J. V. McKelvey, E. E. Moots, E. N. Oberg, Fred Robertson, R. M. Robinson, E. R. Smith, L. W. Swanson, H. P. Thielman, Henry Van Engen, Roscoe Woods, E. A. Zubay.

The following officers were elected for the coming year: Chairman, Professor W. M. Davis, Cornell College; Vice-Chairman, Professor B. E. Gillam, Drake University; Secretary, Professor Fred Robertson, Iowa State College.

Professor Hildebrandt delivered an invited address at 4:00 P.M. on Friday. Five of the following papers were read Friday afternoon, and the remainder Saturday morning.

1. *A use of matrices in accounting*, by Professor E. S. Allen, Iowa State College, introduced by the Secretary.

Let an organization have n service departments $S_1, \cdots, S_n$, acquiring respective surpluses $s_1, \cdots, s_n$; let the fraction of s_i attributable to aid given by S_α be $l_{\alpha i}$, the fraction attributable to producing department P_α be $m_{\alpha i}$ ($\sum_{\alpha=1}^{n} l_{\alpha i} + \sum_{\alpha=1}^{p} m_{\alpha i} = 1$). If the producing departments $P_1, \cdots, P_p$ are to receive the total surplus $s_1 + \cdots + s_n$, an equitable distribution will be

$$P = M(1 - L)^{-1}S,$$

P, M, L, S being the matrices of the corresponding lower case letters.

2. *Simultaneous reduction of quadratic forms*, by Professor B. Vinograde, Iowa State College, introduced by the Secretary.

This paper will appear in the *Proceedings of the Iowa Academy of Science.*

3. *Linear recurrence relations between iterated kernels*, by C. E. Langenhop, Iowa State College.

A kernel of an integral equation, whose iterates satisfy a linear relation

$$a_1K_1(x, y) + a_2K_2(x, y) + \cdots + a_nK_n(x, y) = 0, \qquad a \leqq x, y \leqq b,$$

with $a_1 \neq 0$, is a solution of the integral equation $K(x, y) = \int_a^b H(x, t)K(t, y)dt$, where $H(x, t)$ is another linear combination of the iterates. If $K(x, t)$ is bounded, $H(x, t)$ is bounded, and there are then only a finite number of linearly independent $K(x, y_i)$ where $a < y_i < b$, $i = 1, 2, \cdots, n$. Thus $K(x, y)$ must be of the special form $\sum_{i=1}^{n} \alpha_i(x)\, \beta_i(y)$. For a kernel of this form it is known that the Fredholm determinant $D(\lambda)$ is a polynomial. The converse is not true, but if the Fredholm first minor $D(x, y; \lambda)$ of a bounded $K(x, y)$ is a polynomial in λ, and if $D(\lambda)$ is a polynomial of the same degree, then the iterated kernels satisfy a linear relation as above, and $K(x, \beta)$ is of the special form. If $K(x, y)$ is continuous and symmetric, then it is sufficient for the $D(\lambda)$ merely to be a polynomial that $K(x, y)$ to be of the special form given above.

4. *A third order irregular boundary value problem*, by H. E. Ellington, State University of Iowa, introduced by the Secretary.

Consider the differential system

$$\frac{d^3u}{dx^3} + [\rho^3 p(x) + q(x)]u = 0,$$

with boundary conditions $u(0) = u'(0) = u(\pi) = 0$. The transformation $u(x) = h(x)\, R(z)$, $z = \phi(x)$ will reduce the equation to the form

$$\frac{d^3R}{dz^3} + \phi(z)\frac{dR}{dz} + \left[\frac{\phi'(z)}{2} + \psi(z)\right]R + \rho^3 R = 0$$

with the boundary conditions $R(a) = R'(a) = R(\pi) = 0$. The characteristic solutions are

$$R_n(z) = -2e^{\rho_n(z-a)/2}\left[\cos\left\{\frac{\pi}{3} + \frac{\sqrt{3}}{2}\rho_n(z-a)\right\} + \delta_n(z)\right]$$

where $|\delta_n(z)| < \delta$ if $n < N$ and $a \leqq z \leqq \pi$.

5. *Construction of parabolas*, by Masihur Rahman, Iowa State College, introduced by the Secretary.

Mr. Rahman showed ways of constructing parabolas and (or) discovering some of their properties by use of compass and straight edge. As one illustration he found the point of intersection of a parabola and a line through the focus with given direction, without the usual procedure of constructing the parabola.

6. *Instructional aids for the teaching of junior college mathematics*, by Professor E. H. C. Hildebrandt, Northwestern University.

Professor Hildebrandt exhibited a number of charts and models. He discussed devices of interest to both mathematics majors and other students. After the address the audience inspected the models and discussed their use with Professor Hildebrandt.

7. *An approximate solution of a differential equation by difference equations*, by Spencer Macy, Iowa State College, introduced by the Secretary.

The theory of approximation by difference equations developed by R. G. D. Richardson is extended to a linear self-adjoint equation

$$[p(x)u''(x)]'' + [q(x)u'(x)]' + [r(x) + \nu k(x)]u(x) = 0 \tag{1}$$

where γ is a parameter. Quadratic forms Q_1 and Q_2 can be found such that $Q_1 = \gamma Q_2$ for the minimum value of Q_1. The set of homogeneous linear equations derived by minimizing Q_1 with respect to

$u(x_i)$ with the constraint $Q_2=1$ is used to find a linear form corresponding to the integral equation equivalent to (1). A method of minimization of Q_1 due to G. H. Shortley is used on the equation of a stiff vibrating string to find the first and second characteristic values.

8. *Mapping of a region on the unit circle by orthogonal polynomials*, by R. N. Goss, Iowa State College, introduced by the Secretary.

In 1921 Szego (*Math. Zeits.*, vol. 9, pp. 218–270) proved that a region in the complex plane bounded by a closed continuous rectifiable curve is mapped on the unit circle $|G(t)| \leqq 1$ by the function

$$G(t) = \lim_{n\to\infty} \left\{ \frac{A}{K_n(a,a)} \int_a^t [K_n(a,z)]^2 dz \right\}$$

where $K_n(a, z)$ depends upon a sequence of orthogonal polynomials associated with the bounding curve, and A is a constant. The generality of its application suggested use of the method in the mapping of airfoils. To test the practical characteristics of the mapping method, a region bounded by the minor arcs of two intersecting circles of different radii was chosen, with numerical data compatible with airfoil dimensions. It was found that for this type of region the successive approximations converge slowly, and involve increasingly laborious computation at each stage. It was concluded on the basis of this and other tests that the method can be employed profitably only for regions for which no simpler mapping function is known, and where efficient computing equipment is available.

9. *A note on the relaxation method*, by Professor R. E. Gaskell, Iowa State College.

The relaxation method was applied to a linear system of equations for purposes of illustration, and then to non-linear systems. Some difficulty in convergence may be encountered in non-linear systems. It may be overcome if the residues are not completely liquidated; that is, if the corrections are damped.

10. *A class of integral identities*, by W. H. Marlow, State University of Iowa, introduced by the Secretary.

Mr. Marlow considered families of definite integrals related to the Gamma function by means of the Laplace transform and other transformations.

11. *On the theory of limits in general analysis*, by Professor E. W. Chittenden, State University of Iowa.

The Peano limit of a function on a directed system with values in a topological space is defined and used to establish theorems about iterated limits.

12. *Complete systems of invariants of the cyclic groups of equal order and degree* (by title), by Dr. C. W. Strom, Washington, D. C., introduced by the Secretary.

This paper will be published in the *Proceedings of the Iowa Academy of Science.*

13. *A two-dimensional potential problem with applications to soil drainage*, by Professor Don Kirkham, Iowa State College, introduced by the Secretary.

Professor Kirkham discussed the flow of water in equally spaced rain tiles of uniform depth. These tiles were embedded in one soil medium lying over another of different permeability. He illustrated by sand over clay and vice-versa. He gave a solution to the problem under special conditions.

14. *A general finite difference summation formula*, by P. M. Bailey, State University of Iowa, introduced by the Secretary.

Where the general term $\phi(x)$ of a series satisfies the relation $\phi(x)=f(x+h)-f(x)=\Delta_h f(x)$, various sums of the series are obtainable. The two following formulae giving the definite and indefinite sums of the $\phi(x)$ series, span r, in terms of the function $f(x)$ were presented:

$$\sum_{a=a}^{a+n(-\nu)r}{}_r\ \phi(x) = \sum_{a=0}^{(h/r)-1}{}_1\ \Delta_n f(\alpha + xr)$$

$$\sum{}_r\ \phi(x) = \sum_{k=0}^{(h/r)-1}{}_1\ f(x + kr).$$

The formulae apply when h/r is integral. As an example, the first formula was applied to the summation of a reciprocal factorial function.

FRED ROBERTSON, *Secretary*

APRIL MEETING OF THE LOUISIANA-MISSISSIPPI SECTION

The twenty-fifth annual meeting of the Louisiana-Mississippi Section was held at Southwestern Louisiana Institute, Lafayette, Louisiana, on Friday and Saturday, April 23–24, 1948. Professor Z. L. Loflin, Vice-Chairman for Louisiana, presided at the Friday afternoon session, and Professor T. A. Bickerstaff, Vice-Chairman for Mississippi, presided at the Saturday morning session.

There were one hundred six persons registered, including the following thirty-five members of the Association: T. A. Bickerstaff, Ann Buchanan, H. E. Buchanan, Virginia Carlton, J. C. Currie, Margaret R. Davis, W. L. Duren, Katherine S. Foote, W. W. Gandy, L. M. Garrison, F. C. Gentry, M. E. Mills, A. L. Gilmore, W. C. Griffith, R. V. Guthrie, J. A. Hardin, H. T. Karnes, C. G. Killen, Z. L. Loflin, J. W. McClimans, Dorothy McCoy, A. C. Maddox, B. E. Mitchell, C. V. Newsom, I. C. Nichols, R. L. O'Quinn, P. K. Rees, F. A. Rickey, H. F. Schroeder, H. L. Smith, V. B. Temple, J. F. Thomson, B. A. Tucker, P. M. Tullier, A. D. Wallace.

The following officers were elected for one year: Chairman, W. L. Duren, Tulane University; Vice-Chairmen: Eleanor Walters, Delta State Teachers College, and Peter Tullier, Loyola University; Secretary-Treasurer, F. A. Rickey, Louisiana State University. The 1949 meeting will be held at the University of Mississippi, Oxford, Mississippi.

The following papers were presented at the Friday afternoon session:

1. *The inversion of certain integrals*, by Professor D. B. Sumner, Louisiana State University. (Introduced by Professor P. K. Rees.)

Inversion formulas obtained by Widder and Pollard for the generalized Stieltjes transform are used in a modified form to obtain inversion formulas in the form of a contour integral, which is a generalization of the original result of Stieltjes. The argument is based on a new generalization of Cauchy's "singular integral." The method is capable of extension to a large class of integral equations, in particular, to the convolution transform.

2. *Subpedal curves*, by Professor V. B. Temple, Louisiana College.

The author first defined a pedal curve as the locus of the vertex of a right angle, and gave, as examples, the circle, the ellipse, and the witch of Agnesi. He showed four other pedal curves, namely, the differential axle, the racquets, the cane knife, and the boomerang. He then defined a subpedal curve as the vertex of a right angle projected by the vertex of a right angle. As examples of subpedal curves he showed a few drawings and their equations from his collection. These included, the egg, the fig, the pumpkin, and the longhorns. He further showed that the cissoid of Diocles is a subpedal curve and gave its general equation as $X^2+Y^2=abY^2/(aY+bX)$, where a and b are, respectively, the X- and Y-intercepts of the straight line directrix.

3. *Mathematical topics for college freshmen*, by Professor S. W. Shelton, Northwestern State College. (Introduced by the secretary.)

4. *Dimension and compacting*, by Professor A. D. Wallace, Tulane University.

Herein "space" will mean "normal Hausdorff space." A space X will be said to have dimensional type S provided that for each closed subset A of X and each map $f:A\to S$ there is an extension g of f mapping X into S. For simplicity assume that S is an absolute neighborhood retract. It was shown by Alexandroff and Hurewicz that (assuming X separable metric) X has the dimensional type of the n-sphere if and only if its dimension is at most n. This result has recently been extended by Alexandroff, Dowker and Hemmingsen without the assumption that X be separable metric. They also showed that the dimensions of X and βX (the Chech-Stone-Tychonoff compacting of X) are equal. In this note it is shown that: In order that X have dimensional type S it is necessary and sufficient that βX have dimensional type S.

5. *The solution of Killing's matrix equations*, by Professor M. E. Gillis, Blue Mountain College.

In Killing's equation $UV=(V+1)U$, U and V being matrices, V is given and is assumed to be in the Jordan classical canonical form. Then the most general solution U is exhibited. This form of U has certain submatrices which are zero, while the non-zero submatrices are triangular with all diagonal elements equal. Certain theorems about solutions of the equations were stated.

6. *A condition for the oscillation of the solutions of a system of homogeneous linear differential equations*, by Mr. C. P. Gadsden, Tulane University. (Introduced by Professor W. L. Duren.)

Consider the system (1) $dx/dt=Ax+By$, $dy/dt=Cx+Dy$, where A, B, C, D are continuous functions of t defined on the interval (2) $0\leqq t<\infty$. In the case of constant coefficients A, B, C, D it is known that if $(A-D)^2+4BC<0$, then every solution, $[x(t), y(t)]$, of (1) oscillates; that is, the functions $x(t)$ and $y(t)$ change sign continually as $t\to\infty$. The question may be asked whether a similar condition can be given for the case in which A, B, C, D are not necessarily constants. The following theorem provides an affirmative answer to this euqestion.

Theorem: Let $x(t)$, $y(t)$ be any non-trivial solution of (1). Suppose that for all t and (2) we have $(A-D)^2+4BC<-m<0$, $|B|<M$ and $|C|<M$, where m and M are constants. Then both $x(t)$ and $y(t)$ change sign on the interval $t_1<t<t_1+\Delta t$, t_1 arbitrary on (2), provided Δt is taken sufficiently large.

7. *Recent problems in the training and preparing of mathematics teachers*, by Professor C. R. Trott, University of Mississippi. (Introduced by Professor T. A. Bickerstaff.)

8. *Twenty-five years with the Louisiana-Mississippi Section of the M. A. of A.*, by Professor H. F. Schroeder, Louisiana Polytechnic Institute.

Professor Z. T. Gallion of Southwestern Louisiana Institute presided at a joint banquet with the Louisiana-Mississippi Branch of the National Council of Teachers of Mathematics that was held at Oak Grove Inn Friday evening. Professor C. V. Newsom of Oberlin College, Oberlin, Ohio, was guest speaker for both organizations at the banquet and again on Saturday morning. His addresses were:

1. *Relationship of the Association and the National Council.*

After reviewing recent developments in the field of mathematical education, Mr. Newsom emphasized that many urgent problems could be solved only by the cooperation of college and secondary teachers. In particular, he suggested a study of the entire mathematics curriculum from the first grade to graduate school in the light of new mathematical knowledge that is available, the needs of modern science, and the teaching problems introduced as a result of mass education.

2. *Mathematics and our culture.*

This paper presented some points of view in regard to the significance of mathematics in man's attempt to comprehend his environment. Mr. Newsom expressed the idea that more of the philosophical contributions of mathematics should be a part of all elementary courses in mathematics.

F. C. GENTRY, *Secretary.*

CALENDAR OF FUTURE MEETINGS

Thirty-second Annual Meeting, Columbus, Ohio, December 31, 1948.

Thirty-first Summer Meeting, Boulder, Colorado, September, 1949.

The following is a list of the Sections of the Association with dates of future meetings so far as they have been reported to the Secretary.

ALLEGHENY MOUNTAIN

ILLINOIS, Bradley University, Peoria, May 13–14, 1949

INDIANA, University of Notre Dame, Spring, 1949

IOWA, Drake University, Des Moines, April 15–16, 1949

KANSAS

KENTUCKY

LOUISIANA-MISSISSIPPI, University of Mississippi, Oxford, Spring, 1949

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NEBRASKA, Lincoln, May, 1949

NORTHERN CALIFORNIA, San Francisco, January 29, 1949

OHIO, Ohio State University, Columbus, April 2, 1949

OKLAHOMA

PACIFIC NORTHWEST, Oregon State College, Corvallis, Spring, 1949

PHILADELPHIA

ROCKY MOUNTAIN, Colorado School of Mines, Golden, April, 1949

SOUTHEASTERN, University of Alabama, University, March 18–19, 1949

SOUTHERN CALIFORNIA, John Muir Junior College, Pasadena, March 12, 1949

SOUTHWESTERN

TEXAS, Denton, Spring, 1949

UPPER NEW YORK STATE, University of Buffalo, April 30, 1949

WISCONSIN, Lawrence College, Appleton, May 14, 1949

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